CHAPTER 13

SIMPLE REACTION TIMES AND THE BLONDEL-REY LAW[†]

INTRODUCTION

The process of deriving empirical sensory laws from an entropic or information theoretical base, namely Equation (9.19), will be continued. However, I now deal with those processes that lie at the limits of my understanding. I present matters here as well as I am able, but there are gaps in my comprehension.

We recall that *reaction time* is the time between the onset of a stimulus and the beginning of an overt response. *Simple reaction time* requires that a subject press a key or button immediately upon detection of a stimulus. An excellent introduction to reaction times is given by P. Rabbit (1987). Simple reaction time with Piéron's empirical laws have been introduced in Chapter 3, and the Blondel-Rey (-Hughes) law, together with the Bloch-Charpentier law, were introduced in the same chapter. Now is a good time to review these various experimentally discovered laws, as well as the approximate law of Laffort, Patte, Etcheto and Wright on olfactory thresholds, and the Ferry-Porter law. These various empirical or phenomenological laws may seem to be strange bedfellows, unlikely to be directly related to one another. However, it transpires that the laws are, indeed, closely linked theoretically and mathematically. The same fundamental equation of entropy seems capable of generating all of them with some degree of success.

There have been at least two full books published in the English language that are dedicated to the study of reaction times, one authored by Welford (1980) and the other by Luce (1986). The present chapter has very little in common with those works, emphasizing, as it does, the unification of reaction time phenomena with other sensory effects, and the derivation of general laws of perception, not yet taking into account the matter of interindividual differences.

Cattell (1886) perceived that simple reaction time decreases with increasing stimulus intensity. Thus, for example, it requires less time to react to a bright light than to a dim one. Piéron captured this idea by using a host of empirical equations, one of which is given by Equation (3.26). It might be thought that this phenomenon (decreased reaction time with increased stimulus intensity) could be accounted for by the principle of *energy summation* or *temporal summation / integration*. For example, the intensity of light can be expressed as power, measured in watts which are joules per second. Then the product of intensity with time is equal to the light energy propagated in that time. The light receptor may summate this energy, permitting a reaction when the summated energy reaches some threshold value. There are, however, problems with the energy summation hypothesis. It does not, to my knowledge, lead to graphs of simple reaction time vs. stimulus intensity of the correct form. It will not, for example, allow a derivation of the Piéron laws for simple reaction time (or, at least, I am not aware of such a derivation). Moreover, energy summation does not seem to generalize easily to the chemical senses. It does lead to a crisp explanation for the Bloch-Charpentier law, Equation (3.22), for light and sound, but runs into difficulty with stimuli above threshold.

In the present chapter we replace the principle of energy summation by the principle of information summation, an idea which we have begun to develop in the previous chapter. After exposure to a stimulus, information begins to accumulate within the perceptual unit (a structure we shall formulate progressively). When summated information reaches the threshold value of ΔH, reaction can take place and conscious sensation can occur. This concept of an informational differential threshold, ΔH, is extended from Chapter 12, where it was utilized as the quantity of information needed to discriminate

between two stimuli of different intensities. Now it is to be used as the quantity of information needed to react. Many of these ideas were introduced by Norwich *et al.* (1989). As in Chapter 12, we shall not really require the $F = kH$ relation. Two new concepts of a fundamental nature will be introduced in this chapter: the time delay factor, ξ, and the informational distinction between neuronal and behavioral adaptation processes.

DERIVATION OF AN EQUATION GIVING SIMPLE REACTION TIME AS A FUNCTION OF STIMULUS INTENSITY

We make use of the now-familiar concept, that after presentation of a steady stimulus there ensues an adaptation process, and that as adaptation proceeds, entropy ("potential information") falls, and information is gained. Adaptation registers neurally in the afferent neurons, and adaptation to a stimulus with constant intensity is described by the *H*-function (9.19),

$$H = \frac{1}{2} \ln(1 + \beta I^n / t) = H(I, t) . \tag{13.1}$$

As always, t_o is defined as the time at which H reaches its maximum value, and H decreases for $t > t_o$. Let ΔH be the decrease in entropy between the times t_o and $t_r > t_o$. That is,

$$\Delta H = H(I, t_o) - H(I, t_r) . \tag{13.2}$$

If ΔH is the minimum quantity of information required to make a simple reaction to the stimulus of intensity, I, then t_r is equal to or less than the simple reaction time. We shall speak later about the neuromuscular time lags which effectively guarantee that t_r is less than simple reaction time, but, for the moment, let us ignore this time lag.

Introducing Equation (13.1) into (13.2),

$$\Delta H = \frac{1}{2} \ln(1 + \beta I^n / t_o) - \frac{1}{2} \ln(1 + \beta I^n / t_r) . \tag{13.3}$$

Solving for t_r,

$$t_r = \left(\frac{1}{t_o \, e^{2\Delta H}} - \frac{1 - e^{-2\Delta H}}{\beta I^n} \right)^{-1} . \tag{13.4}$$

t_r is taken as a function of I; ΔH is taken as a constant threshold information.

Interestingly, while t_r looks as if it depends on 4 independent parameters, β, t_o, n, and ΔH, it depends, in fact, on only 3 independent parameters, which may be seen as follows. Let $t_{r\,\text{min}}$ be the minimum possible value of t_r. This minimum value will occur when the denominator on the right-hand side of Equation (13.4) is maximum, which will, in turn, occur when I is maximum. Letting $I \to \infty$ in (13.4),

$$\boxed{t_{r\text{min}} = t_o \, e^{2\Delta H}} . \tag{13.5}$$

Let I_{min} be the minimum value of I for which a response is possible (threshold value of I). As $I \to I_{\text{min}}$, $t_r \to \infty$. That is, for $I = I_{\text{min}}$, the denominator of the fraction on the right-hand side of Equation (13.4) approaches zero:

$$\frac{1}{t_o \, e^{2\Delta H}} - \frac{1 - e^{-2\Delta H}}{\beta I_{\text{min}}^n} = 0 .$$

Solving for I_{min},

$$\boxed{I_{\text{min}} = \left(\frac{t_o(e^{2\Delta H} - 1)}{\beta} \right)^{1/n}} , \tag{13.6}$$

or, equivalently,

$$\frac{t_o \, e^{2\Delta H} (1 - e^{-2\Delta H})}{\beta} = I_{\min}^n , \tag{13.6a}$$

or, introducing Equation (13.5),

$$\frac{(1 - e^{-2\Delta H})}{\beta} = \frac{I_{\min}^n}{t_{r\min}} . \tag{13.6b}$$

Introducing Equations (13.5) and (13.6b) into (13.4),

$$t_r = \left[\frac{1}{t_{r\min}} - \frac{1}{t_{r\min}} \left(\frac{I_{\min}}{I} \right)^n \right]^{-1} ,$$

or

$$\boxed{t_r = \frac{t_{r\min}}{1 - (I_{\min}/I)^n} .} \tag{13.7}$$

Equations (13.4) and (13.7) are mathematically equivalent, but from (13.7) we can see that t_r depends only on *3* independent parameters, $t_{r\min}$, $I_{\min}$, and n.

We note, again, the appearance of the power function exponent, n. The ubiquitous n seems to pervade all equations governing sensory function.

Equation (13.7), then, is put forward as the equation of entropy giving simple reaction time, t_r, as a function of stimulus intensity, I. We see that as I increases in value, t_r becomes smaller, the universally observed relationship between stimulus intensity and reaction time.

Let us return, now, to the thorny issue of the claim that the quantity t_r has upon simple reaction time. Is t_r the simple reaction time? t_r is, by definition, the time taken for the *H*-function, measurable in an appropriate afferent neuron, to decline through a range, ΔH, natural units of information. This adaptation process occurs before reaction to a stimulus can occur. But, presumably, after the information, ΔH, has been transmitted to the brain, a signal must be sent via the motor neurons from brain to muscle, after which the subject can signal her/his response. It would seem reasonable, therefore, to define t_R as the simple reaction time (from stimulus onset to the motor act of pressing a button), and to set

$$t_R = t_r + t_{\text{lag}} . \tag{13.8}$$

t_r is, then, the time for the adaptation curve to register (in sensory neurons) a decline, ΔH, in entropy, and t_{lag} is the delay time taking account of conduction time to the cerebral cortex, through motor neurons and synapses to muscle, and the contraction of muscle. Welford (1980) divides t_R further into four divisions, and Halpern (1986) further analyzes t_r into components that are significant for the human sense of taste. It would seem that t_{lag} is an appreciable interval of time, so that

$$t_R > t_r . \tag{13.9}$$

If we introduce Equation (13.8) into (13.7),

$$t_R = \frac{t_{r\min}}{1 - (I_{\min}/I)^n} + t_{\text{lag}} , \tag{13.10}$$

giving t_R, a measurable quantity, as a function of I, with 4 independent parameters, the previous three plus t_{lag}. However, all attempts to curve-fit Equation (13.10) to simple reaction time data, using values of t_{lag} that are greater than zero, have failed. Various investigators in different laboratories have tried to provide such a curve-fit, using simple reaction times to visual, auditory and gustatory stimuli, but none has succeeded. Values obtained for the parameters are non-physiological (t_{lag} or n negative) and are inconsistent between different sets of data. On the other hand, when t_{lag} is taken to be zero, as in Equation (13.7), the results are usually quite satisfactory, as will be shown below. It is as if conduction

BOX 13.1 An approach to the problem of conduction time

Suppose that neuronal conduction time is introduced by means of a multiplicative factor, rather than by an additive term as in Equations (13.8) and (13.10):

$$t_R = t_r + t_{\text{lag}}. \tag{B1}$$

That is, we let

$$t_o \text{ (neuronal)} = \xi t_o \text{ (simple reaction time)} \tag{B2a}$$

and

$$\beta \text{ (neuronal)} = \xi \beta \text{ (simple reaction time)}, \tag{B2b}$$

where $0 < \xi < 1$. Then t_o (s.r.t.) and β (s.r.t.) are the values of t_o and β obtained by measuring motor or efferent responses in simple reaction time (s.r.t.) (Tables 13.1 and 13.2), while t_o (neuronal) and β (neuronal) are the values of these parameters as they would emerge from analysis of adaptation data obtained from afferent neurons. When I is dimensionless, that is, measured in units relative to threshold, both t_o and β have the dimensions of time, so that Equations (B.2a) and (B.2b) suggest that there is a change in *time scale* between the *neuronal station* and the *behavioral station* (afferent neuron and simple reaction time).

The advantages of the transformation (B.2a) and (B.2b) are

(i) In explaining that $t_{r\,\text{min}}$ (*neuronal*), as obtained by Equation (13.5), becomes $\xi t_o\, e^{2\Delta H}$, which is less than $t_{r\,\text{min}}$(s.r.t.), which makes sense due to conduction delays.

(ii) In showing why an equation of the form of (13.7) fits simple reaction time data so well despite conduction time delays. That is, replacing t_o by ξt_o in Equation (13.5) results in $t_{r\,\text{min}}$ being replaced by $\xi t_{r\,\text{min}}$ in Equation (13.7). $\xi t_{r\,\text{min}}$ is simply curve-fitted as a single parameter. The delay is "built into" the parameter, so to speak.

(iii) In showing why I_{min} (neuronal), as obtained from Equation (13.6), remains the same as I_{min}(s.r.t.) when t_o and β are replaced by ξt_o and $\xi \beta$ respectively, which makes sense, because minimum intensity for reaction should be independent of any time lags.

Since only $t_{r\,\text{min}}$ and not I_{min} changes under the transformation, Equation (13.7) is transformed into

$$t_r \text{(neuronal)} = \frac{\xi t_{r\,\text{min}}}{1 - (I_{\text{min}}/I)^n} = \xi t_r \text{(s.r.t.).} \tag{B3}$$

Equations (B.2a) and (B.2b) are, then, equations postulated purely *ad hoc*, in order to account for a neural conduction time lag. They may be of use in converting time-dependent parameters t_o and β measured in an experiment on simple reaction time for use in a situation where conduction lag does not evidently occur.

The value of ξ will vary depending on which station is used to determine t_o and β. For simple reaction time, I am guessing that $\xi \simeq 0.5$. That is, about 50% of the minimum reaction time is due to neuromuscular conduction time and 50% is due to the neuronal adaptation process. This value of ξ seems to give tolerable results when used with simple reaction times to visual stimuli and sodium chloride taste stimuli. Psychophysical or behavioral adaptation (decrease in subjective magnitude) also lags considerably behind neuronal adaptation. I put it to the reader to suggest a value for ξ for behavioral adaptation relative to neuronal adaptation.

The disadvantage to the use of Equation (B.2a) and (B.2b) is that the resulting Equation (B.3) is not readily interpretable in terms of current understanding of the physiology of the nervous system. Moreover, since β is increased for reaction time by Equation (B.2b), therefore, apparent sampling rate is slowed. If fact, all time-dependent processes are slowed as we go from receptor to behavior, resulting in a spreading out of the curve of simple reaction time along the time-axis – a kind of relativistic time dilatation. Rosenblith and Vidale (1962, Figure 14) observed this phenomenon in responses to auditory stimuli. Behavioral response times (reaction times) spanned at least 100 to 300 milliseconds, cortical response times spanned about 10 milliseconds (see text as well as Figure 14), while earliest electrical responses in the auditory nerve spanned only about *2* milliseconds. There is a mystery here, and Equations (B.2a) and (B.2b) only transcribe the mystery into mathematical language. This "time dilatation" with approximate preservation of the amount of information transmitted is demonstrated explicitly in Figures 14.1a and 14.1b.

in motor neurons occurred with infinite speed, which is, of course, absurd. It is particularly bizarre because Helmholtz introduced the measurement of reaction time specifically as a means of measuring neuronal conduction time, precisely the element that is here being ignored. The mystery may be linked to the findings of Libet (1985), as discussed by Norwich *et al.* (1989).

Equation (13.7) fits simple reaction time data very well (the reader may wish to look ahead at Figures 13.1 and 13.2). Because it is highly unlikely that neuronal conduction time can really be ignored, and because an additive t_{lag} as in Equation (13.10) just does not agree with measured data, I suggest, tentatively, the modification shown in Box 13.1. This modification would allow for a time lag due to conduction time, while preserving the algebraic form of Equation (13.7). The "true" or neuronal values of t_o and β are diminished from their values as measured in reaction time data, using a multiplicative reduction or diminution factor, but the diminution factors are identical. The true or neuronal values are given by Equation (B.2) and (B.2a). All is not roses with this altered formulation, but it is the best I can suggest at the moment.

In the analysis that follows, we shall use Equation (13.7) with t_r identified with the measured simple reaction time. However, if we attempt to obtain from reaction time data a value for t_o that is compatible with data from sensory neurons, we shall probably have to use a diminution factor, ξ, as suggested in Box 13.1.

DERIVATION OF ONE OF THE EMPIRICAL EQUATIONS OF H. PIÉRON

Piéron carried out extensive experimentation on simple reaction times for many sensory modalities. He fitted his data to many equations of convenience (Piéron, 1914, 1920). Frankly, I haven't even attempted to derive most of these algebraic forms from the informational equation. However, the one empirical equation that he presented in his text (Piéron, 1952) is easily derived from Equation (13.7), so I present the derivation here.

Piéron's empirical equation for simple reaction time is (Equation (3.26))

$$t_r = CI^{-n} + t_{r\,\text{min}} , \tag{13.11}$$

where C is constant. To derive it, we consider the case where

$$(I_{\text{min}}/I)^n \ll 1 . \tag{13.12}$$

The denominator of the fraction on the right-hand side of Equation (13.7) can then be expanded in a binomial series:

$$[1 - (I_{\text{min}}/I)^n]^{-1} \simeq 1 + (I_{\text{min}}/I)^n + \ldots \tag{13.13}$$

retaining only the first order term. Hence, Equation (13.7) becomes

$$t_r \simeq t_{r\,\text{min}}[1 + (I_{\text{min}}/I)^n] .$$

That is,

$$t_r = (t_{r\,\text{min}}.I^n_{\text{min}})I^{-n} + t_{r\,\text{min}} , \tag{13.14}$$

which is identical with Piéron's Equation (3.26)/(13.11). Moreover, we can identify Piéron's constant, C, with the constant $t_{r\,\text{min}} \cdot I^n_{\text{min}}$. However, we must not forget that the derivation was based upon the approximation (13.12).

THE CONSTANT ΔH FOR SIMPLE REACTION TIME

We do not know from *a priori* consideration the value of ΔH, the minimum quantity of information necessary to react. However, I am prepared to put forward a conjecture based on a suggestion made several years ago by L. M. Ward in a personal communication. The conjecture is that ΔH, the minimum

quantity of information required to react to a stimulus, and, hence, the absolute informational threshold for detection of the stimulus, is equal to H_{max}, the channel capacity for that stimulus, as measured in a *neuronal* adaptation process. That is, ΔH for reaction = (*magical number*) $\simeq$ ln(6) = 1.75 natural units, or about 2.5 bits. I am making this conjecture because, as we shall see in the coming pages, it permits us to make accurate predictions for the senses of taste, vision and hearing. However, the conjecture will force us, for the first time, to make a definite distinction between neuronal and behavioral adaptation processes.[1]

EVALUATION OF THE INFORMATIONAL EQUATION FOR SIMPLE REACTION TIME

We proceed now to evaluate Equation (13.7) for two modalities: audition and vision. Later (see "Compiling a Common Set of Parameters for Taste of Sodium Chloride") we shall also evaluate Equation (13.7) for the sense of taste of sodium chloride.

(i) *Audition*.

The data of Chocholle (1940) were analyzed. Subjects were requested to press a button as soon as possible after a tone was sounded.[2] The subjects were tested over a large range of sound intensities and over a wide range of frequencies. For a given frequency, reaction time, of course, was observed to decrease with increasing stimulus intensity. Chocholle's data for a 1000 Hz tone are listed in Table 13.1. I is given in relative units of sound pressure (lowest pressure = 1.00).

The data were fitted to Equation (13.7) using the least squares criterion, and the following parameter values were obtained:

$$t_{r\,min} = 0.117 \text{ s} \tag{13.15}$$

$$I_{min} = 0.449 \tag{13.16}$$

$$n = 0.439 \tag{13.17}$$

Table 13.1   Data of Chocholle, Subject I, 1000 Hz tone

Sound Pressure	Reaction time(s) measured	Reaction time(s) theoretical
1.00×10^5	0.110	0.117
3.20×10^4	0.110	0.118
1.00×10^4	0.112	0.118
3.20×10^3	0.118	0.119
1.00×10^3	0.124	0.121
320	0.129	0.124
100	0.139	0.129
31.6	0.148	0.138
10.0	0.161	0.157
3.16	0.192	0.203
2.51	0.218	0.220
2.00	0.248	0.243
1.58	0.276	0.275
1.26	0.312	0.320
1.00	0.398	0.394

Note: Theoretical values were calculated from Equation (131.8).

With these parameter values in place, Equation (13.7) becomes

$$t_r = \frac{0.117}{1 - \dfrac{0.449^{0.439}}{I^{0.439}}} = \frac{1}{8.55 - 6.01\,I^{-0.439}} \ . \tag{13.18}$$

Chocholle's data together with the fitted function, (13.18), are shown in Figure 13.1. We note that the values for $t_{r\min}$ and $I_{\min}$ are nominal, falling a little below the smallest measured values for t_r and I respectively. The value obtained for n, the power function exponent, is 0.439, close to, but less than, the value of about 0.6 expected from psychophysical experiments. A similar result was reported by Marks (1974).

Using the conductive delay factor suggested in Box 13.1, $\xi = 0.5$,

$$t_{r\min} \text{ (neuronal)} = \xi\, t_{r\min}(\text{s.r.t.}) = (0.5)(0.117) = 0.059 \text{ s}, \tag{13.19}$$

using Equation (B.3). Alternatively, we can write from purely neuronal considerations, $t_o = 0.002$ s (from Yates *et al.*, 1985, Figure 5), and $\Delta H = 1.75$ natural units, using the "channel capacity" conjecture, so that

$$t_{r\min} \text{ (neuronal)} = t_o\, e^{2\Delta H} = (0.002)\, e^{(2)(1.75)} = 0.066 \text{ s}. \tag{13.20}$$

(ii) *Vision*

The data of Doma and Hallett (1988) were used. Subjects were required to track a target visually. The latency between the time the target moved and the time the eye moved was taken as a measure of reaction time. Subjects were tested over a range of light intensities,[3] for light of various wavelengths. Doma and Hallett's data for yellow-green light (654 nm) are given in Table 13.2. Again, the intensities are in relative units.

The data were fitted to Equation (13.7), which provided the following parameter values:

$$t_{r\min} = 0.149 \text{ s} \tag{13.21}$$

$$I_{\min} = 0.0332 \tag{13.22}$$
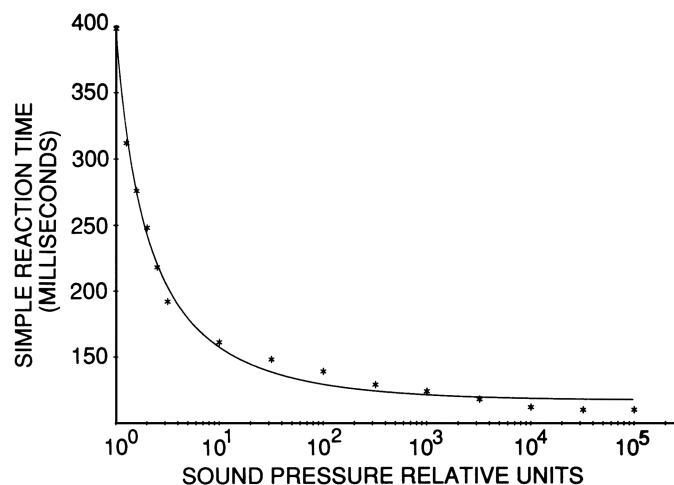
$$n = 0.288 \ . \tag{13.23}$$

Figure 13.1 Data of Chocholle (1940) (Subject 1, 1000 Hz tone). Simple reaction time to an auditory stimulus. The data are listed in Table 13.1. The smooth curve was generated by Equation (13.18).

Table 13.2 Data of Doma and Hallett, Yellow-green light, 564 nm.

Intensity of light	Reaction time (s) measured	Reaction time (s) theoretical
100	0.163	0.165
31.6	0.173	0.173
10.0	0.185	0.184
3.16	0.208	0.203
1.00	0.239	0.238
0.794	0.244	0.248
0.631	0.262	0.260
0.501	0.277	0.274
0.398	0.291	0.291
0.316	0.305	0.312
0.251	0.335	0.337
0.200	0.364	0.369
0.158	0.414	0.411
0.126	0.475	0.476
0.100	0.543	0.547

Note: Theoretical values were calculated from Equation (13.24).

When these parameter values are inserted into Equation (13.7), we obtain

$$t_r = \frac{1}{6.71 - 2.52 I^{-0.288}} \; . \tag{13.24}$$

Doma and Hallett's data and the fitted function (13.22) are shown in Figure 13.2.

Again, the parameters $t_{r\,\mathrm{min}}$ and I_{min} take on proper values, just below the corresponding smallest measured values for t and I respectively. The value of n is 0.288, very close to the commonly quoted value of 0.3.

We observe that the power function exponent, n, appears both in Equation (13.7) and in its approximation, Piéron's equation (13.11) and (13.14). People have often mused about the enigmatic appearance of an exponent of about the magnitude of the "Stevens" exponent appearing in an equation for simple reaction time. We can now understand why it appears in this position.
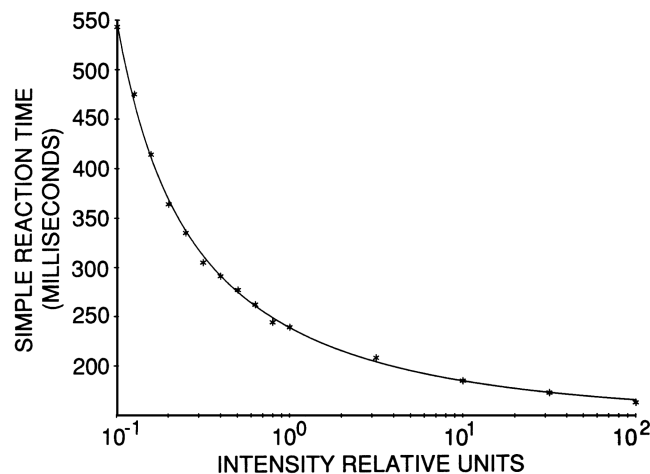

Figure 13.2 Data of Doma and Hallett (1988) (Yellow-green light at 564 nm). Simple reaction time to a visual stimulus. The data are listed in Table 13.2. The smooth curve was generated by Equation (13.24).

THE INFORMATIONAL BASIS OF REACTION TIMES

The above discussion is, of course, predicated on the assumption that a fixed quantity of information, ΔH, must be transmitted before a subject can react. Note that evaluation of two parameters, $t_{r\,min}$ and I_{min}, does not permit us to solve for unique values of the three parameters, β, t_o, and ΔH. Nonetheless, as we shall see in our forthcoming analysis of the Blondel-Rey law, there is good evidence that the same set of parameters will describe both types of experiment. We also find below, that at least for the sense of taste of sodium chloride, we seem fairly close to having a single set of parameter values that will describe and predict all experimental findings.

Here, as is the case with the many other sensory equations developed in this book, no allowance is made for interindividual differences. Rather, a single equation is derived to represent a typical, or perhaps averaged, result.

Please recall that here, as elsewhere, we are utilizing a kind of conservation law for information. When ΔH units of information have been collected, a response is possible, etc. No mechanism for the mediation of reaction is provided. The shorter reaction time observed for a more intense stimulus follows from the properties of the H-function. Referring to Equation (13.1),

$$\frac{\partial H}{\partial t} = \frac{-\beta/(2t^2)}{1/I^n + \beta/t} \, , \tag{13.25}$$

so that for a given t, the larger I becomes, the greater is the value of the derivative $|\partial H/\partial t|$, and, hence, the more rapidly does H decline. *That is*, *stimuli of greater intensity transmit information at a greater rate than stimuli of lower intensity.*

The history of information theory as a means of explaining reaction time began some time ago when information theory was young. Hick (1952) analyzed the *choice reaction time* (cf. simple reaction time). A subject is required to make one selection from among m choices, for example by pressing one lit key from among 10 keys. The choice reaction time was found to be proportional to $\log(m + 1)$, or, approximately, to the information required to press the correct key, giving rise to *Hick's law*. Hyman (1953) and Hellyer (1963) showed that the time required for a subject to react to a complex task is a linear function of the number of bits of information involved in the task. Colin Cherry, in his fascinating book (1957) describes and illustrates an experiment demonstrating Hick's law that the reader can try for herself / himself. The experiment is attributed to J.C.R. Licklider. A simpler experiment is suggested by Coren and Ward (1989), Demonstration Box 2-3. We can watch Hick's law at work by analyzing the data of Merkel (1885), as reported by Coren and Ward, p. 39. Merkel's data are plotted in Figure 13.3. It is seen that choice reaction time increases as the number of response
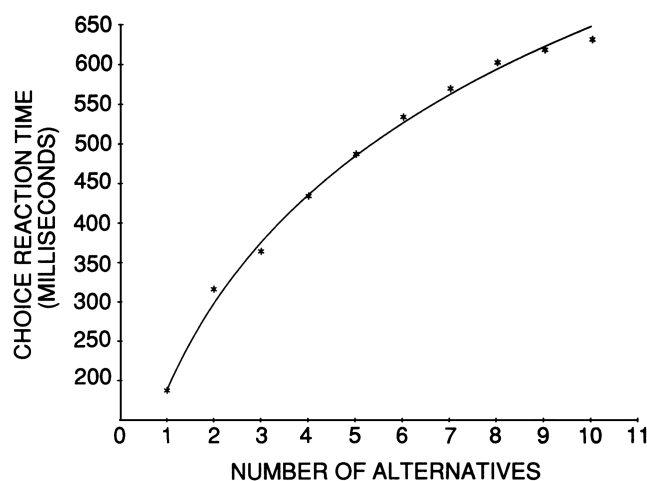
Figure 13.3 Data or Merkel (1885), demonstrating Hick's law. Choice reaction time is plotted against number of alternatives. The smooth curve is the logarithmic function given by Equation (13.26): if m is the number of alternatives, choice reaction time varies as $\log(1 + m)$.

alternatives increases. The smooth curve was generated by the fitted function

$$\text{choice reaction time} = 270.74 \, \ln(1 + m) \, , \tag{13.26}$$

where m is the number of response alternatives.

An interesting application of information theory is found in the papers of E.T. Klemmer (1956, 1957). Klemmer showed that simple reaction time varies with the subject's uncertainty *about the time of stimulus occurrence*.

Our present use of information in the analysis of simple reaction times differs from the above. We generate simple reaction time curves based on the assumption that ΔH units of information are required for a reaction, where H is a function purely of stimulus intensity. In the quantized view of stimulus signals, more intense signals not only present greater uncertainty about the mean value of the signal, but yield or give up their uncertainty more rapidly upon sampling than do less intense signals (Equation (13.25)). In summary,

Hick: Subjects react more rapidly to simpler tasks because less information is required to make a decision.

Norwich: Subjects react more rapidly to more intense stimuli because information can be collected more rapidly.

COMPILING A COMMON SET OF PARAMETERS FOR TASTE OF SODIUM CHLORIDE

Let us examine the constant, β, as it appears in Equation (9.19)/(13.1). The term $\beta I^n / t$ is added to unity, so that it is a dimensionless quantity. That is, β has the dimensions of t / I^n. When intensity, I, is measured in relative units, it, too, is dimensionless, and in this case β will have the dimensions of time. However, when I is measured in more common laboratory units, such as moles per liter, the dimensions of β become more complicated (Appendix). In this section, we shall work with simple, relative units of concentration of sodium chloride, so that β will have the dimensions of time, and, specifically, be in units of seconds. There is no curve-fitting in this section. We are trying to establish a *plausible* set of parameter values.

There is evidence that the "time scale" or "time range" of adaptation-dependent processes increases as one proceeds from the primary sensory afferents, through cortical events, to the behavioral or psychophysical report of the subject. For example, an adaptation process that requires, say, 60 milliseconds to go to completion in an auditory ganglion cell, may require 60 seconds to go to completion behaviorally. Such a change in time scale is allowed for the the ξ-factor ($0 < \xi < 1$) introduced in Box 13.1. That is, t_o and β, as measured, say, in an adaptation process in a ganglion cell, become t_o / ξ and β / ξ respectively in a behavioral (psychophysical) adaptation process (decreasing subjective magnitude with increasing time). Conversely, we can use the ξ-factor to convert the value of t_o found in a (behavioral) experiment on simple reaction time to a neuronal t_o-value

$$t_o(\text{neuronal}) = \xi \, t_o(\text{behavioral}) \tag{13.27a}$$

and

$$\beta(\text{neuronal}) = \xi \, \beta(\text{behavioral}). \tag{13.27b}$$

For time in general,

$$t(\text{neuronal process}) = \xi \, t(\text{behavioral process}) \tag{13.28}$$

The result of such a transformation of variables is to leave invariant the quantity of information transmitted by an adaptation process measured at any station between primary afferent neuron and muscle effector (*e.g.* vocal activity providing psychophysical report), which may be seen mathematically as follows. Between time, t_o, and any time, t,

$$H(\text{neuronal}) = \tfrac{1}{2} \ln(1 + \beta \, I^n / t_o) - \tfrac{1}{2} \ln(1 + \beta \, I^n / t) \, . \tag{13.29a}$$

Using Equations (13.27) and (13.28)

$$H(\text{behavioral}) = \frac{1}{2}\ln(1 + \xi\beta I^n/(\xi t_o)) - \frac{1}{2}\ln(1 + \xi\beta I^n/(\xi t)) . \qquad (13.29b)$$

The latter two equations are identical since ξ cancels in Equation (13.29b), therefore,

$$H(\text{neuronal}) = H(\text{behavioral}). \qquad (13.30)$$

That is, we measure the same quantity of information from a neuronal process as from a behavioral (psychophysical) one. Moreover, from Equations (13.27a) and (13.27b),

$$\beta(\text{neuronal}) / t_o(\text{neuronal}) = \beta(\text{behavioral}) / t_o(\text{behavioral}). \qquad (13.27c)$$

That is, the ratio $\beta{:}t_o$, whether measured neuronally or behaviorally, will be constant. The above approach will suffice for current purposes, but for further work we must seriously regard the matter of note 1, wherein the informational content of the two adaptation processes can differ.

Let us turn our attention now to ΔH, which is an informational threshold. We learned in Chapter 12 that $\Delta H = \Delta F / k$ corresponded to that constant quantity of information transmitted with a change in subjective magnitude of one jnd. While ΔH was constant for each subject, the magnitude of ΔH was not fixed, but varied with the criterion used by the investigator to *define* the jnd. In the case of the Weber fraction for the taste of sodium chloride solution measured by Holway and Hurvich, we found $\Delta H = 0.126$ natural units.

The value of ΔH required for reaction to a stimulus, or the absolute detection of a stimulus, need not be equal to a quantity of arbitrary magnitude determined by the method of the investigator. Rather it can be determined by an average or mean, independent of any criterion imposed by the investigator (other than pressing the button). We postulated above that ΔH for absolute detection is approximately equal to the entire information capacity of the modality, or the channel capacity, represented by $H_{\max}$. As we have seen, this value has been cited as about 2.0 bits for taste, 2.5 bits for audition, etc.[4]

Finally, then, the proposed universal values for taste of sodium chloride.

There is very wide variation in the values of n cited for taste of sodium chloride, extending from about 0.5 to 1.4. I have selected here $n = 1.0$, but I encourage the reader to experiment with other values of n. Other parameter values will change as n is changed.

Accordingly, ΔH (absolute detection) $\simeq 2.0$ bits $\rightarrow 2\ln 2 = 1.39$ natural units of information. ΔH for discrimination of the jnd was taken as 0.126 natural units, the value we used in Chapter 12. There is, of course, an arbitrary element to ΔH(jnd), as discussed earlier.

The problem is, then, to find a value for t_o(neuronal), which is the time following stimulus onset at which a neuronal adaptation curve will have its maximum amplitude (action potentials per second). Pfaffmann (1955) studied the response of neurons in the chorda tympani of the cat, dog and rat, and reported: "The activity of the chorda tympani initiated by applying taste solutions to the tongue is typically an asynchronous discharge of impulses. For an electrolyte like sodium chloride, the latency of the discharge is of the order of 30 msec; ..." This value, however, is too great to permit the informational calculation of simple reaction times that are in accordance with the measurements made by Bujas for human beings in 1935. A value of $t_o = 0.015$ s or smaller was necessary; $t_o = 0.015$ s was adopted.

By inspection, because it tends to give the right answers, I selected $\beta/t_o = 15$, when intensity, I, is measured in units relative to threshold = 1. Therefore, $\beta = 0.225$ s.

A proposed universal set of parameters for the taste of sodium chloride is, then

$$n = 1.0 \qquad \beta = 0.225 \text{ s} \qquad t_o = 0.015 \text{ s}.$$

$$\Delta H(\text{absolute detection}) = 1.39 \text{ n.u.}$$

$$\Delta H(\text{discrimination of jnd}) = 0.126 \text{ n.u.} \qquad (13.31)$$

Using this parameter set, I submit, we can generate most, if not all, of the sensory functions of tasted sodium chloride solutions. Let's check it out.

(i) *Computing the value of the Weber constant*:

From Equation (12.6), the value of the Weber constant is given by $2\Delta H/n$. Substituting from (13.31) values for ΔH and n,

$$\text{Weber constant} = (2)(0.126)/1.0 = 0.25 .\tag{13.32}$$

This value compares favorably with the approximate value obtained from Holway and Hurvich's data, as shown in Figure 12.3.

(ii) *Computing the equation relating simple reaction time to stimulus intensity:*

From Equation (13.5), dividing by ξ to go from neuronal to behavioral,

$$t_{r\,\min} = (t_o/\xi)\,e^{2\Delta H} = (0.015/0.5)\,e^{(2)(1.39)} = 0.48 \text{ s.}\tag{13.33}$$

From Equation (13.6),

$$I_{\min} = \left(\frac{(t_o/\xi)(e^{2\Delta H} - 1)}{\beta/\xi} \right)^{1/n} .$$

ξ cancels, so that

$$I_{\min} = \left(\frac{0.015\,(e^{(2)(1.39)} - 1)}{0.225} \right)^{1/1.0} = 1.0 \text{ relative units.}\tag{13.34}$$

When the above values for $t_{r\,\min}$ and $I_{\min}$ are inserted into Equation (13.7), the resulting curve can be compared with the experimental data of Bujas (1935). The predicted curve is not bad at all. A better fit to the data is obtained by making a small adjustment in the values of the parameters (best made manually rather than by least squares) so that

$$t_{r\,\min} = 0.342 \text{ s,}\tag{13.33a}$$

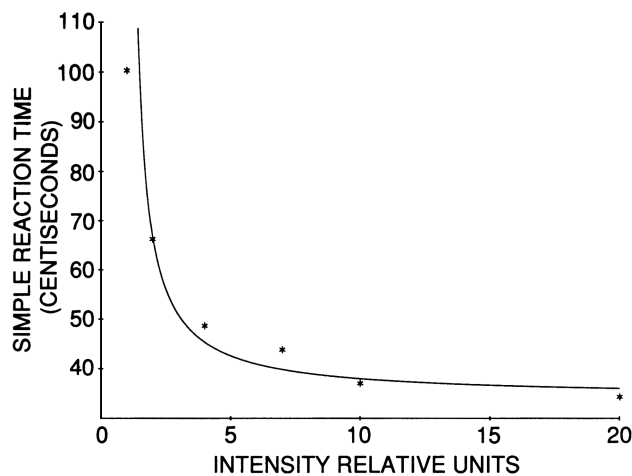$$I_{\min} = 0.98 \text{ relative units.}\tag{13.34a}$$

Figure 13.4 Data of Bujas (1935). Simple reaction time to a gustatory stimulus (sodium chloride solution). Intensity of the stimulus is plotted in concentration units relative to threshold = 1. The smooth curve is plotted using Equation (13.7) with $t_{r\,\min} = 0.342$ s and $I_{\min} = 0.98$ units, very close to the values obtained from the universal parameter set for taste of sodium chloride, Equation (13.31). The smooth curve has *not* been curve-fitted to the data by a least squares method (please see main text).

The resulting curve, together with the data of Bujas are plotted in Figure 13.4. Notice that Equation (13.7) cannot be properly fitted to the data by a least squares criterion *applied to the ordinate only* (that is, computing sums of squares of errors using "errors" in simple reaction time only, and ignoring "errors" in stimulus intensity), because of the extreme sensitivity of the curve-fit to the data point with the lowest intensity. Tiny errors in the abscissa of this data point changed the parameters of the curve-fit greatly.

Further analyses of Bujas' data have been given by Norwich (1991).

(iii) *Calculating the Weber fraction as a function of stimulus intensity:*

From Equation (12.6),

$$\Delta I / I = (2\Delta H / n)\left(1 + \frac{1}{(\beta / t) I^n}\right). \qquad (13.35)$$

Substituting Holway and Hurvich's $t = 10$ s, as well as values for β, ΔH and n from (13.31),

$$\Delta I / I = \frac{(2)(0.126)}{1.0}\left(1 + \frac{1}{(0.225 / 10) I^{1.0}}\right). \qquad (13.35a)$$

If the above function is evaluated for $I = 1$ (threshold) to $I = 160$ (about 4 M solution), the result is an approximation to the findings of Holway and Hurvich. The theoretical curve actually falls somewhat more rapidly than Holway and Hurvich's curve (Fig. 12.2), similar to the curve of $\Delta I / I$ for sucrose observed by Lemberger (Fig 3.5a). If the reader wishes to check the match of the theoretical curve to Holway and Hurvich's data, I recommend changing concentration to molar units in the manner shown in the Appendix. Threshold $\simeq 0.025$ M so that $\lambda = 40$. However, we are lucky, indeed, to even have an approximation here. We have used the neuronal value of β (What value of ξ should be used?), and have utilized Equation (12.6), a mode II equation, to describe Holway and Hurvich's experiments which were, in fact, conducted according to mode I (refer to Chapter 12, "On the physical meaning of ΔI").

(iv) *Calculating the information transmitted by an adaptation process over t seconds:*

Transmitted information for a hypothetical neuronal adaptation process is given by

$$H = \frac{1}{2}\ln(1 + \beta I^n / t_o) - H_{min}. \qquad (9.19) / (13.1)$$

Setting $I =$ some intermediate value for concentration, say 10 relative units (corresponding approximately to 0.1 M), and selecting values for β and n from (13.31),

$$H = \frac{1}{2}\ln(1 + (0.225)(10)^{1.0} / 0.015) - H_{min}$$

$$= 2.51 - H_{min} \text{ natural units.}$$

I do not know, *a priori*, the value of H_{min} for neuronal adaptation to a sodium chloride solution, so I can go no further. A value of $H_{min} \simeq 1.3$ n.u. would leave H at about the right value. That is, one would expect that the steady state firing rate of the neuron would be about one-half of its maximum firing rate.

(v) *Calculating the subjective magnitude as a function of time in an adaptation process:*

Let us re-cast the adaptation data of Gent and McBurney, that we analyzed in Chapter 11, into relative concentration units. The 0.32 M solution becomes 12.8 in units relative to threshold ($\simeq 0.025$ M). The value of n obtained by Gent and McBurney for sodium chloride for their subjects was 0.5. Then we must select the value of 12.86 seconds for β, so that βI^n becomes $(12.86)(12.8)^{0.5} = 46$.s, which is the value we obtained by curve-fitting (see Chapter 11). If $\beta = 15 t_o$ (see above Equation (13.31)), then $t_o \simeq 1$ second for the taste of sodium chloride. Psychophysically, taste intensity curves tend to rise for a few seconds (*i.e.* taste intensity builds for this time interval), so the calculated value of t_o is, perhaps, tolerable.

Again, we are not curve-fitting in this section. Rather we are trying to establish a plausible set of values characterizing the sense of taste of sodium chloride in all its manifestations, and the set suggested by Equation (13.31), with allowance made for the range of measured values of *n*, does seem to merit consideration.

THE LAWS OF BLONDEL AND REY, OF HUGHES, AND OF BLOCH AND CHARPENTIER[††]

While the intensities of stimuli for reaction time are at and above threshold, the intensities of stimuli used to demonstrate the law of Blondel and Rey, etc. are precisely *at* threshold. "How long," they asked, "must a stimulus of intensity I' be held so that the stimulus is just perceptible?" However, despite this shift in the scale of intensity values, from the point of view of the entropy theory, the Blondel-Rey and its associated laws utilize the same variables and parameters as those used to define simple reaction time: stimulus intensity, I, duration of stimulus, t, as well as β and n, t_o and ΔH. It seems reasonable that ΔH, the minimum quantity of information required to perceive a stimulus in a Blondel-Rey experiment, would be the same quantity, ΔH, needed to react in a simple reaction time experiment, since if you can detect the stimulus you can react to it. However, there is room for error in reasoning here.

Anyway, algebraically, the Blondel-Rey equation should be exactly the same as the equation for simple reaction time, with the terms rearranged, since all variables and parameters are the same. Here the unifying capacity of the entropy equation is in clear evidence. Let us just rearrange the terms in Equation (13.7) while introducing (13.5):

$$(I_{\min}/I)^n = 1 - t_o e^{2\Delta H}/t_r . \tag{13.36}$$

In keeping with the nomenclature of Equation (3.25), let us replace t_r by t, and $I_{\min}$ by I_∞ (which has the same meaning), and I by I_{thresh}. Then

$$I_\infty / I_{\text{thresh}} = (1 - t_o e^{2\Delta H}/t)^{1/n} \tag{13.37}$$

or

$$I_{\text{thresh}} / I_\infty = \left(1 - t_o e^{2\Delta H}/t\right)^{-1/n} . \tag{13.38}$$

When

$$t \gg t_o e^{2\Delta H} , \tag{13.39}$$

we can expand the right-hand side of Equation (13.38) in a binomial series and drop terms of order higher than the first:[5]

$$\boxed{I_{\text{thresh}} / I_\infty = 1 + t_o e^{2\Delta H}/(nt) .} \tag{13.40}$$

If we now set the constant, $t_o e^{2\Delta H}/n$ equal to a, we have

$$I_{\text{thresh}} / I_\infty = 1 + a/t , \tag{13.41}$$

which is the algebraic form of the Blondel-Rey law, Equation (3.25).

We have not really completed the derivation of the Blondel-Rey law (and its auditory analog which we have called "Hughes' law"), because the variable, t, which appears in Equation (13.41) represents time since stimulus onset, while t in Equation (3.25) represents the duration of the flash. These two times are not equal, the former, time since stimulus onset at which information, ΔH, is transmitted via a *neuronal* adaptation function, will be greater than the latter, the duration of the flash. Therefore, the Blondel-Rey constant has been derived to be *not less than*

$$a = t_o e^{2\Delta H}/n \tag{13.42}$$

$$= t_{r\,\min} / \text{ power function exponent.} \tag{13.43}$$

That is, the Blondel-Rey constant, in our unified system of equations for sensation, is approximately equal to the ratio of the minimum reaction time to a visual signal to the power function exponent ("Stevens exponent") for vision. We can test the theory partially for at least one set of data, the simple reaction time data of Doma and Hallett (Table 13.2). Using the value of $t_{r\,min} = 0.149$ from Equation (13.21) multiplied by the conduction factor, $\xi = 0.5$ (taking us from behavioral to neuronal adaptation), and power function exponent for a point source on dark background = 0.5 (Coren and Ward, Table 2.11), we have from Equation (13.43)

$$a = (0.5)(0.149)/0.5 = 0.149 \text{ s,} \tag{13.44}$$

or, from first principles, t_o(neuronal) $\simeq 0.002$ s (value from audition[6,†††]); $\Delta H = 1.75$ n.u.;

$$a = t_o\,e^{2\Delta H}/n = 0.13 \text{ s.} \tag{13.45}$$

Since t (stimulus duration) $< t$ (since onset) $= bt$, where $b < 1$, "true a" $= a / b > a$, referring to Equation (13.41). The range of values measured for the Blondel-Rey constant is given by Williams and Allen (1977, p. 43) as 0.055 to 0.35, so the values calculated by Equations (13.44) and (13.45) are in accord with the measured values.

Now, we may recall from Chapter 3 that Blondel and Rey observed that their law *contained* the law of Bloch and Charpentier. That is, in Equation (13.41), when t is very small, the second term on the right-hand side dominates, so that

$$I_{min}/I_\infty = a/t \,,$$

which is the Bloch-Charpentier law. Therefore, since we have *derived* Equation (13.41), we can do the same thing, thereby also deriving the Bloch-Charpentier law. Right?

Well, almost right. Don't forget that we have used inequality (13.39) in order to derive (13.41). If we let t become too small, that is, if we use stimulus durations that are too brief, we shall violate this inequality. Again, taking $t_o \simeq 0.002$ s. and $\Delta H = 1.75$ n.u., we have $t_o\,e^{2\Delta H} = 0.066$ s, so that by (13.39) we must have $t > 0.066$ s, and preferably $t \gg 0.066$ s for validity of our derivation, while the Bloch-Charpentier law is, apparently valid experimentally for flashes much briefer than 0.066 s. Again, we must make allowance for the fact that $t > t_{flash}$.

THE BLONDEL-REY LAW AND THE FERRY-PORTER LAW

We have seen in the previous section how we were able to transform the equation for simple reaction time into the Blondel-Rey law by simple algebraic manipulation. When the variables are the same and the parameters are the same, the laws must be mathematically identical. We shall try the same trick with the Ferry-Porter law, but we are in deep water – perhaps too deep. We deal here not with a single stimulus, but with multiple, sequential stimuli (flashes), which the entropy equation, (13.1), was not constructed to handle. Between flashes, there is a process of dark adaptation, which is beyond our theoretical grasp at this moment. We must continue to deal with the variable, t, which is the time since stimulus onset rather than the flash duration. And we shall attempt to derive the curve for pure cone vision in the central fovea, although we lack definitive knowledge of the relevant ΔH-values. However, the game can be fun anyway. If you would like to follow me where angels would, no doubt, fear to tread, please read on.

Again, a change in nomenclature. Let us represent I_{thresh} simply as I.

Then, Equation (13.40) becomes

$$I/I_\infty = 1 + t_o\,e^{2\Delta H}/(nt)\,. \tag{13.46}$$

When

$$t_o\,e^{2\Delta H}/(nt) \ll 1 \tag{13.47}$$

(cf. Equation (13.39)), then, using a Taylor series, we can approximate the right-hand side of Equation

(13.46) by an exponential function:

$$I/I_\infty = \exp\left[\left(\frac{t_o\,e^{2\Delta H}}{n}\right)\frac{1}{t}\right].$$ (13.48)

Taking logs of both sides,

$$\ln I - \ln I_\infty = \left(\frac{t_o\,e^{2\Delta H}}{n}\right)\frac{1}{t}\,,$$

or

$$(1/t) = (n/t_o\,e^{2\Delta H})\ln I - (n/t_o\,e^{2\Delta H})\ln I_\infty\,.$$ (13.49)

Ignoring, for the moment, the distinction between t and flash duration, we write, for a flashing light with equal times for "on" and "off,"

$$\text{frequency} = 1/(2t)$$ (13.50)

$$\text{critical fusion frequency [CCF]} = (n/2t_o\,e^{2\Delta H})(1/\log_{10}e)\log_{10}I$$
$$- (n/2t_o\,e^{2\Delta H})\ln I_\infty\,.$$ (13.51)

This equation, then, does give the correct algebraic form of the Ferry-Porter law,

$$\text{CFF} = c_1\log I + c_2,$$ (3.30)

and provides theoretical values for the constants, c_1 and c_2.

Informationally, Equation (13.51) states that a minimum time, t, must pass in order that a flash of intensity, I, may transmit a quantity of information, ΔH. If this criterion is met, the flash can be discriminated, or seen "crisply." If the duration of time is less than t, the minimum "quantum" of information, ΔH, will not be transmitted, and the flash either will not be visible or will merge with the next flash. A complete informational treatment of the Ferry-Porter law must take account not just of a single flash, but of the repetitive sequence of flashes and the process of dark adaptation between flashes. Such analysis is beyond the current scope of the theory. A complete theory must also take into account the distinction between duration and time since onset.

The slope of the straight line defined by Equation (13.51), obtained by plotting CFF against $\log_{10}I$, can be evaluated in one of two ways: $t_o\,e^{2\Delta H}$ can be evaluated from reaction time data, or from first principles.

(i) Using data from simple reaction time with Equations (13.5) and (13.21),

$$t_o\,e^{2\Delta H} = \xi t_{r\,\min} = (0.5)(0.149) = 0.0745 \text{ s}.$$

The ξ carries us from behavioral to neuronal. Again taking $n = 0.5$, the slope

$$(n\,/\,2t_o\,e^{2\Delta H})(1/\log_{10}e) = 7.73 \text{ s}^{-1}\text{(s.r.t.)}$$ (13.52a)

(ii) From first principles, $t_o\,e^{2\Delta H} = 0.002\,e^{(2)(1.75)} = 0.066$ s.
so that

$$(n\,/\,2t_o\,e^{2\Delta H})(1/\log_{10}e) = 8.72 \text{ s}^{-1} \text{ (first principles)}.$$ (13.52b)

Hecht (1934) suggested that the observed values for slope tend to cluster about 11 s^{-1} for images on the central fovea (no statistics were given), so the theoretical values are quite close.

We notice that, in theory as well as by experiment, the slope of the straight line in the Ferry-Porter law plotted using common logs is approximately equal to the reciprocal of the Blondel-Rey constant (comparing Equations (13.40) and (13.51) and noting that $2\,\log_{10}e \simeq 1$).

Again, here as elsewhere, we are plagued by the necessity of merging the results of experiments that are not quite compatible. We should value the results of a study on simple reaction times to light restricted to a narrow beam on the fovea, or 5 degrees peripheral to the fovea, in order to calculate the slopes of Ferry-Porter plots made for the same retinal locus.

We observe that Equation (13.51) is valid, strictly speaking, only for values of t permitted by inequality (13.47); that is, t of the order of 0.13 s or greater (Equation (13.45)), corresponding at most to 3 Hz (!?). However, the Ferry-Porter law is observed experimentally to be valid for frequencies as high as about 50 Hz. That is, the theoretical equation, (13.51) is valid over a range of frequencies much greater than we had any legitimate reason to expect. The theoretician's problem is usually quite the opposite of this: one can often not find experimental verification over the range of values for which an equation was derived. Resolution of this paradox may lie in the same quarters as before: $t_{flash} < t = t_{neuronal}$.

The above theoretical derivation of the Ferry-Porter law is very brief, requiring only about one page of development beyond the Blondel-Rey law. The reader is reminded, however, that this a kind of conservational derivation, dealing with limits upon the rate at which information from a photon beam can be transmitted to a photoreceptor. A good deal of study has been made, and is being made, of the excellent papers of E. Hisdal, in an effort to improve the theory as it is presented here. The brevity of the theory, as put forth above, may be been taken to imply that the author is ignoring the truly prodigious number of scientific papers dealing with flickering lights and the mechanism of flicker fusion phenomena. These papers have not been ignored: the diffusion theory of Ives (1922), the photochemical theory of Hecht and Verrijp (Hecht and Verrijp, 1933; Hecht, 1934, 1937), the multiple-stage models of Kelly (1961), and many more recent efforts. The reader is reminded that the informational approach *does not compete* with these models of mechanism; it *complements* the models of mechanism. The informational approach provides restrictions or guidelines governing the development of mechanisms of any perceiving system. *Mechanisms* of perception and sensation must develop (evolve?) within the limitations imposed by information transfer.

BRIGHTNESS ENHANCEMENT

"Brightness enhancement" refers to "an increase in the brightness of an intermittent light over that of a steady light of the the same luminance" (Graham, 1965). I have not even attempted to treat this phenomenon mathematically. Let me just remind the reader that within the informational or entropy theory, *variance* of light samples, rather than their mean, is the determinant of brightness. Therefore, a flickering light should, at appropriate frequencies of flicker, indeed, appear brighter than a steady light of the same luminance, since the flicker can in principle increase the variance of samples from the light beam.

OLFACTORY THRESHOLDS

To conclude this chapter, we make a very approximate derivation of what is, in fact, a very approximate law, given by Equation (3.29). Taking common logs of both sides of Equation (13.6) and multiplying through by n, gives

$$n \log_{10} I_{min} = \log_{10}[t_o(e^{2\Delta H} - 1)/\beta] \, . \tag{13.53}$$

Is there any reason to think, now, that the quantity of the right-hand side of this equation should be approximately constant for all odorants? We could argue the matter on theoretical terms, but I think the result would be inconclusive. The best one can say at the moment is that *if* $t_o(e^{2\Delta H} - 1)/\beta$ is constant and equal to K, then

$$-n \log_{10} I_{min} = K \, . \tag{13.54}$$

Using the definition of p_{ol} from Equation (3.28), we have

$$(n)(p_{ol}) = K \, , \tag{13.55}$$

which provides a derivation of Equation (3.29), the approximate law discovered experimentally by Laffort, Patte and Etcheto, with support by Wright.

CONCLUSIONS

In this chapter, we have pushed the entropy theory to its current limits to derive, among other sensory phenomena, simple reaction time as a function of stimulus intensity, the Blondel-Rey-Hughes law, the Bloch-Charpentier law, and the Ferry-Porter law (in part only).

One of the promising results of this chapter, I think, is the set of parameters characterizing the taste of sodium chloride (Equation (13.31)). A time-lag factor, ξ, may have to be employed (or a more cleverly formulated device that will allow for conduction delays), but, by and large, this parameter set has permitted us to account, in a quantitative way, for the results of nearly all sodium chloride taste experiments known to this author that involve a single, pure NaCl stimulus. It will be instructive to see if this data set will be capable of predicting the results of experiments not yet performed or unknown to this author. The theory presented does not seem to be capable of explaining the increase in simple reaction time with increasing taste stimulus duration reported by Kelling and Halpern (1983).

Perhaps the chief complexity introduced in this chapter relates to time scales. There were two distinct problems:

(i) Relating the time scale of behavioral events (such as simple reaction times) to the time scale of neuronal events (such as the adaptation in impulse rate at a ganglion cell). These scales have been related by means of the factor ξ, introduced in Box 13.1. The use of this factor is not completely satisfactory from the theoretical point of view, but it does seem to work.

(ii) Relating the time scale of neuronal events to the time scale of stimulus events (that is, time since onset to duration of stimulus). One must be mindful of the lessons taught in this regard by Ward (1991) psychophysically, and by Wasserman and Kong (1974) neurophysiologically. Nonetheless, the algebraic forms of the laws of Blondel-Rey, Bloch-Charpentier, and Ferry-Porter have emerged, *complete with theoretical derivations of their respective constants*, when "time since onset" replaces "duration of stimulus" in the respective equations.

As we press the variables I and t through their complete physiological range of values, the question of the constancy of the parameters must be examined. If the parameters n, or β were found to be functions of I, the formulation of the *H*-function as it was given in Chapter 9, would have to be reexamined. Whether t_o is a function of I is a moot point. Burke *et al.* (1987) found psychophysically for sodium chloride, quinine sulfate and citric acid stimulation of the anterior tongue that "Higher intensity ratings were associated with faster onset times ...," while Travers and Norgren, recording electrophysiologically in the nucleus of the solitary tract in rats (1989, Figure 2), seemed to show the opposite for sodium chloride stimulation of the anterior tongue and nasoincisor ducts.

We are approaching the end of our experimental evaluation of the entropy theory. Only some brief remarks on the exponent, n, remain for the next chapter. Although we have encountered the current limits to the theory in this chapter, one should not forget the extraordinarily wide range of observed sensory effects that have been captured by the theory: General sensory principles expressed by the law of sensation, the principle of adaptation, the Weber fraction and simple reaction time; and the derivation of a host of special empirical laws, which we shall not enumerate here.

APPENDIX: CHANGING THE UNITS OF THE PARAMETER β

Since β occurs always in the combination $\beta I^n / t$, and $\beta I^n / t$ is dimensionless [occurring, as it does, in $\log(1 + \beta I^n / t)$], the dimensions of β are those of t / I^n. That is, if we alter the units in which either I or t are measured, we must alter the value of the constant, β, accordingly.

Consider particularly a change in the units of I from I to I'. Suppose the change in units is governed by the constant, λ, so that

$$I' = \lambda I .$$

Then,

$$\beta I^n = \beta'(I')^n = \beta'(\lambda I)^n = (\beta' \lambda^n) I^n .$$

That is,

$$\beta' = \beta / \lambda^n .$$

For example, for taste of sodium chloride, with *n* chosen as 1.3, if *I* is measured in molar units, and *I'* is measured in units relative to threshold = 1 unit, then $\lambda \simeq 100$ (since threshold is approximately 0.01 M). Thus,

$$\beta' = \beta / 100^{1.3} .$$

That is

$$\beta \text{ (with } I \text{ in relative units)} = 0.0025 \, \beta \text{ (with } I \text{ in molar units)}.$$

β (with *I* in relative units) will have the dimensions of time.

NOTES

1. The adoption of ΔH for reaction $= H_{max}$ compels us to make a distinction between the neuronal and behavioral adaptation processes, which we have hitherto regarded as "equivalent," adopting the symbol *F* to represent both. When we analyze behavioral adaptation data, such as those of Gent and McBurney (1978), we find examples of perceptible stimuli that transmit fewer than H_{max} bits of information, which would not be possible if H_{max} were the threshold for absolute detection. Hence my insistence that H_{max} units of information be transmitted by the corresponding *neuronal* adaptation curve. Thus we have postulated a distinction between the two types of adaptation process: the neuronal process transmits a minimum of H_{max} units of information, while the behavioral process may reflect the transmission of less information. Evidence in support of this position may be found in Figures 11.6 and 11.7, which each depict neuronal adaptation curves for stimuli of 3 different intensities. In each of the three curves of Figure 11.6, $(F_{max} - F_{min})$ has about the same value. Similarly, in each of the three curves of Figure 11.7, $(F_{max} - F_{min})$ has about the same value. That is, the curves do not adapt to extinction. Since information, ΔH, is equal to $(F_{max} - F_{min})/k$, therefore, approximately the same quantity of information is transmitted by the three stimuli of different intensities that generated each of the three curves. The three curves have different amplitudes, signifying different stimulus intensities, but transmit nearly the same quantity of information.

2. There are many variations in the way experiments to measure simple reaction time are carried out. For example, ready signals my indicate an impending stimulus (Kohfeld, 1969; Botwinick and Storandt, 1972).

3. The term "light intensity" is being used rather glibly here, where proper photometric units should be used. Preferable in most instances would be *illuminance* (millilamberts) or, perhaps *retinal illuminance* (Trolands).

4. There are at least two problems that issue from the use of the absolute informational threshold, ΔH. One is that the threshold intensity, I_∞, cannot, strictly speaking, be determined by eliciting an infinite reaction time. It can be determined, classically (*e.g.* Galanter, 1982), as the weakest stimulus that can still be detected ("reacted to") 50 percent of the time. Modulating factors can be allowed for using signal detection theory. Alternatively, one can use *neural-quantum theory* (e.g. Stevens, 1961, pp 806-813). The second problem deals, perhaps, with even more profound issues. The two informational thresholds, ΔH (discrimination of jnd), and ΔH (absolute detection), have been found to differ substantially in magnitude of information. However, we can argue that ΔH (absolute detection) is, in a sense, the first or lowest jnd – that is, it is the amount of information needed to discriminate the smallest perceptible signal from the zero signal. Why should this jnd require a greater quantity of information than subsequent jnd's? There is something unique about the "first" jnd which I cannot, at this moment, understand.

5. We note that Equations (13.40) and (13.41) are independent of the parameter β, depending only on the parameters *n*, t_o and ΔH.

6. We note that in writing Equations (13.44) and (13.45) no distinction has been made between the parameters characterizing photopic and those characterizing scotopic vision.
——————————— 2003 ed. notes:

†. Simple reaction time, has, perhaps, undergone the greatest modification during the 11 years, 1992 to 2003. While maintaining the same general form for t_r, the parameter, β, has vanished, permitting us to evaluate the critical information "quantum", ΔH, in several ways. The new value differs from the old, so that the set of parameters characterizing the taste of sodium chloride will

change accordingly. However, the philosophy of approach to simple reaction time remains intact: one can react when he/she has received at least ΔH units of information.

††. The derivation of the laws of Blondel and Rey and of Ferry and Porter can now be carried out somewhat more simply and briefly. The extent and effects of approximation can be shown more clearly. The derivation of the Ferry-Porter law has been improved considerably, and insight into the meaning of this law has been enhanced. The new derivation leads to a somewhat different algebraic form for the law. However, again, I retain the original derivations in this second edition of *ISP*.

†††. Not correct. I discovered several years after *ISP* had been published that the value of t_0 for vision seemed to be somewhat greater than 0.002. As mentioned above in note †, the value of ΔH has also been changed. So this entire calculation must be corrected. I retain the original calculations here, as the best that could be done in 1992-3. The same correction will have to be applied to Equation (13.52b): both t_0 and ΔH will have revised values.

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