CHAPTER 3

THE EMPIRICAL LAWS OF SENSATION AND PERCEPTION

In this chapter, we set down the weighty ballast of philosophy and information theory, and examine the somewhat lighter matter of the empirical rules of sensation and perception. By *empirical* laws we mean (Webster's dictionary) laws "making use of, or based on, experience, trial and error, or experiment, rather than theory or systematized knowledge." For somewhat more than one hundred years, beginning (probably) with the work of Weber, empirical laws of sensation have been formulated. These algebraic rules, based essentially on laboratory observations, relating only occasionally to each other, and not derived theoretically from laws in other sciences, have dominated the scientific literature. Each empirical law stands as a universe unto itself: it is neither derived from any simpler principle, nor does it lead to the generation of other laws. Each law has absolute dominion over its own territory. Such is the state of scientific polytheism that we now describe.

The reason for introducing these laws early in the book is that they provide, so to speak, grist for the mill. We shall endeavor, as the informational theory of sensation is developed, to provide theoretical derivations for all of these empirical laws. It is probably better to introduce them earlier and in a group, rather than later as they are invoked.

Some of the empirical laws carry the names of their originator; some, such as "the exponential decay" of this or that quantity are just rules of thumb. We are not concerned with all of these rules, but only a subset of them. In particular, we shall be interested in those empirical laws that govern the relationship between three fundamental variables: I, the steady intensity of a stimulus; t, the time since onset of the stimulus, or, occasionally, the duration of this stimulus; and F, the perceptual variable related to the stimulus. F, you will recall, was defined in Chapter 2. As mentioned in the Introduction to Chapter 1, all stimuli with which we shall be concerned here are steady, or constant stimuli, given in the form of a step function (Figure 1.1). While stimuli that vary with time are of very definite interest, for example those that may vary sinusoidally, their formal treatment is more difficult within this informational or entropic theory, and such progress as has been made with these stimuli will not be reported here. Neither do we grapple with the effects of multiple stimuli that are applied concurrently. So we shall not deal, for example, with the sweetness of a solution of two types of sugar, or with the effect of a masking sound on a pure tone.

With these restrictions in mind, let us examine eight types of experiment performed by physiologists, psychologists and physicists that give rise to well-known empirical equations of sensation. In each case in which the perceptual variable, F, occurs, recall from Chapter 2 that it can be interpreted both psychophysically, as a subjective magnitude (e.g. brightness), and physiologically, as a rate of impulse propagation in a neuron. In Chapter 13, we shall begin to distinguish mathematically between these two interpretations.

THE LAW OF SENSATION

When F is interpreted psychophysically, this law is sometimes referred to as "the psychophysical law."

Ernst Heinrich Weber (1795 — 1878) (pronounce *Vay'ber)* was a German physiologist who was professor of anatomy and later of physiology at Leipzig (Gregory and Zangwill, 1987). He drew attention to the ratio $\Delta I/I$, where ΔI is the smallest difference between two stimulus intensities that can be discriminated. Wrote Weber (in translation):[1] "...in observing the difference between two magnitudes, what we perceive is the ratio of the difference to the magnitudes compared" (Drever, 1952). Together with Fechner, he asserted, after much experimentation on lifting weights,

$$\Delta I / I = \text{constant}, \tag{3.1}$$

where the constant is known as *Weber's constant*. We call the empirical law (3.1) "Weber's Law."

Gustav Theodor Fechner (1801 — 1887) (pronounce $Fech' - ner$, *ch* as in *loch*) was a German physicist who is remembered largely for his work *Elemente der Psychophysik* (1860). Fechner augmented Equation (3.1) by equating the constant on the right-hand side with ΔF, a just noticeable difference in sensation (The F is my symbol, not Fechner's). More specifically, the equation attributed to him is

$$\Delta I / I = \Delta F / a \tag{3.2}$$

where both a and ΔF are constants. Implicit in Equation (3.2) is that a variable, F, can legitimately be defined to quantify human sensation or feeling. Assigning numerical measure to a sensation is rather an audacious suggestion. Equation (3.2) then asserts that if the physical magnitude of a stimulus is changed by ΔI, where ΔI is the smallest change detectable by a human subject, then the corresponding change in sensation, ΔF, will always be constant. The just noticeable difference is abbreviated to *jnd*. Fechner's argument is often stated as follows.

If ΔI and ΔF are small changes in I and F respectively, they may be replaced in Equation (3.2) by dI and dF respectively

$$dI / I = dF / a\,. \tag{3.3}$$

Integrating both sides of Equation (3.3),

$$F = a \ln I + b, \qquad b \text{ constant}, \tag{3.4}$$

or

$$F = a' \log I + b\,, \tag{3.4a}$$

where a' is constant and the logarithm may be taken to any convenient base, say 10. That is, when F is plotted against the logarithm of I, the result expected is a straight line. This is *Fechner's law*, or the *Weber-Fechner law*. A discussion of "Fechnerian integration" and related topics is given by Baird and Noma (1978, Chapter 4). The problem of measuring the quantity, F, is very taxing and has occupied psychophysicists for many years. This problem is discussed in an introductory manner by Coren, and Ward (1989), and in more detail by Baird and Noma (1978). More about early attempts to quantify sensation, and about the legitimacy of Fechnerian integration is given in the first chapter of Marks' book (1974).

The result of Fechner's law is that if "suitable" measure for F can be found, a graph of F against the logarithm of I (to any base) is expected to produce a straight line. That is, for a given modality, plotting experimental values of F against the corresponding values for the logarithm of I should give the result that the data points lie on a straight line whose slope is a' and whose y-intercept is b. Fechner's law might be called a *semilogarithmic law*, because data array linearly when F is plotted on a linear scale while I is plotted on a logarithmic scale. There are many examples in the published literature of measured data that conform to Fechner's law.

While F was interpreted psychophysically by Fechner, we recall again from Chapter 2 that F may also be interpreted neurophysiologically as the frequency of impulses in a sensory neuron. In the well-known paper by Hartline and Graham (1932) also cited in Chapter 2, a single light receptor (omatidium) from the horseshoe crab (*Limulus*) was dissected out together with its primary afferent neuron. The receptor could be stimulated with light of varying intensity, I, and the resulting impulse frequency, F, in the attached neuron, measured. The authors showed that Fechner's law was obeyed. There is evidence, therefore, that Fechner's law, a form of "the law of sensation" is valid to a degree using either interpretation of the perceptual variable, F.

Plateau was a Belgian physicist, contemporary with Fechner. He is often given credit for conceiving the *power law of sensation*, an alternative to the semilogarithmic law of Fechner (Plateau, 1872). This power law is given explicitly by Equation (3.7) below. F. Brentano (1874) attempted to derive this power law beginning from Weber's law. In the twentieth century, this power law of sensation found its most enthusiastic exponent in the person of S. S. Stevens. While I did state above, admittedly, that each of these empirical laws remained independent of all other such laws, there have, nonetheless, been attempts made over the years to link them together. I think that these attempts at deriving the laws are laudable and I present some of them in these pages. Usually, however, it was found necessary to add *other* empirical relations in order to complete the derivation.

The derivation of the Plateau-Brentano-Stevens power law of sensation beginning from Weber's law is attributed to Brentano (Stevens, 1961) and proceeds as follows. Suppose that both the physical magnitude of the stimulus, I, and the subjective magnitude of the stimulus, F, both obey Weber's law. Then from Equation (3.1),

$$\Delta I / I = c_1 ,$$

and

$$\Delta F / F = c_2 ,$$

where c_1 and c_2 are both constants. Then by combining these equations,

$$\Delta I / I = (1/n) \Delta F / F , \tag{3.5}$$

where $n = c_2 / c_1$.

Equations (3.1) and (3.5) are, of course, quite different. Again replacing the finite differences by their corresponding differentials,

$$dI / I = (1/n) dF / F .$$

Assuming the legitimacy of Fechnerian integration, we obtain

$$\ln F = n \ln I + \ln k , \tag{3.6}$$

where $\ln\ k$ is a constant of integration. This equation can be converted immediately into the form

$$F = kI^n . \tag{3.7}$$

This power law of sensation stands in contrast to Fechner's semilogarithmic law. It is important to observe that by taking logarithms *to any base* of both sides of Equation (3.7) we obtain

$$\log F = n \log I + B , \tag{3.8}$$

where B is constant. That is, the Plateau-Brentano-Stevens law might be described as a *full logarithmic law* (cf. Fechner's semilogarithmic law) because data are expected to array themselves along a straight line when both F and I are plotted on logarithmic scales. The slope of this straight line is the power function exponent, n.

Many modalities of sensation have been analyzed by means of the power law (3.7), and characteristic values of the exponent, n (or rather, characteristic ranges of values for n), have been tabulated for each modality. For example, for the intensity of sound, $n \simeq 0.3$ for tones of 1000 Hz. For a complete list of exponents governing the various modalities the reader is referred to the texts, such as Coren and Ward (1989). Stevens spent many years demonstrating that when F was measured by the process of "magnitude estimation" (a free-wheeling assignment of numbers to match the magnitudes of human sensations) the power law (3.7) was the law of best fit. To capture the day, Stevens (1970) re-plotted the *Limulus* data of Hartline and Graham of 1932 on a log-log graph (that is, he made a full logarithmic plot rather than a semilogarithmic plot as Hartline and Graham had done), and what emerged was a straight line as impressive as that obtained by Hartline and Graham. Surely, then, the power law of sensation was "the correct" law of sensation and could lay claim to the title of the psychophysical law!

So, indeed, it may be, but feeling that Fechner might desire a posthumous reply, I played the inverse game. I selected data measured by Stevens (1969) for the sense of taste of saltiness (sodium chloride solution). Stevens had showed from a plot of the logarithm of magnitude estimation vs. the

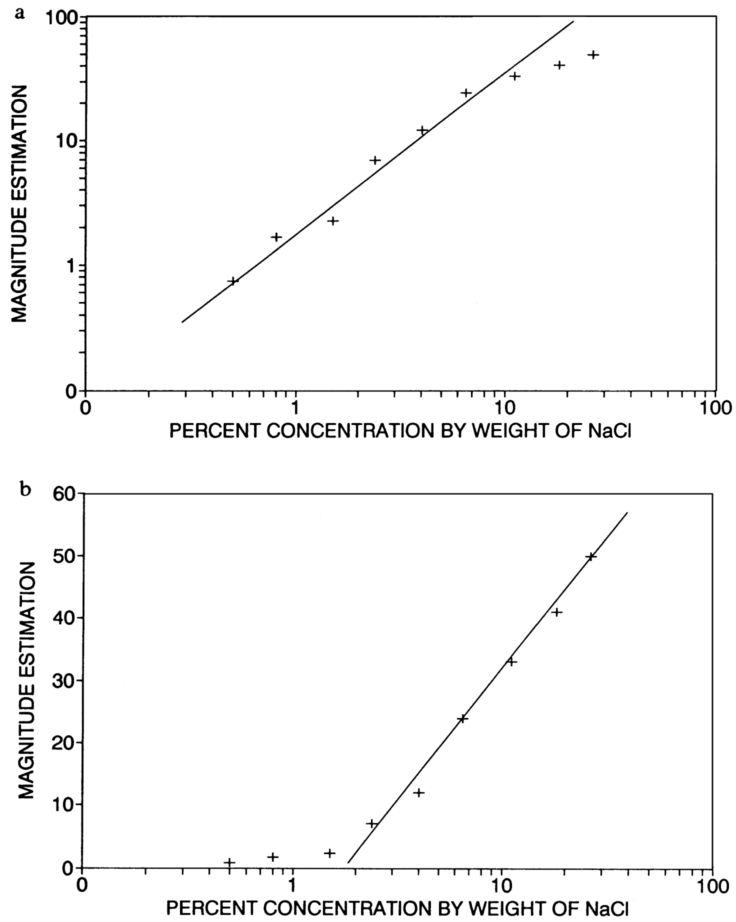

Figure 3.1 a&b (a) Data of S.S. Stevens (1969). Magnitude estimation of the taste of saltiness of solutions of sodium chloride of different concentrations. In this log-log plot, the data are seen to fall nearly on a straight line, except for the two or three *most* concentrated solutions, whose magnitude estimates fall *below* the straight line. This is the type of graph preferred by Stevens. (b) The same data shown in Figure 3.1a are plotted here in a semilog plot. Note that the data again fall very nearly on a straight line, except for the two or three *least* concentrated solutions, whose magnitude estimates fall *above* the straight line. This is the type of graph preferred by Fechner.

logarithm of concentration of solution that these data strongly supported the power law of sensation. In Figure 3.1 the same data are plotted in a Fechner semilogarithmic graph: magnitude estimation (not its logarithm) is plotted against the logarithm of concentration of the solution. The result is quite a decent straight line, thereby affirming Fechner's law!

Is it possible that the two forms of the law of sensation given by Equations (*3.4*) *and* (*3.8*), *Fechner's law and the Plateau-Brentano-Stevens law*, *are really mathematically equivalent over some range of I-values*? To this question we shall certainly return. In the interim, the reader interested in pursuing Fechner vs. Stevens is referred to the very scholarly review of the subject by L. Krueger (1989), or to Krueger's more condensed review (1990).

We have been using the perceptual variable, F, with both the psychophysical and the physiological interpretations. In this regard, one must take note of the papers of G. Borg and his colleagues (for example Borg *et al*., 1967). Taking advantage of the fact that in the human being, sensory nerve fibers mediating taste from the anterior two-thirds of the surface of the tongue pass backwards toward the brain in the nerve called the *chorda tympani*, and that this nerve is surgically accessible as it passes through the middle ear, Borg *et al*. carried out a series of experiments. Two days before surgery was to be performed on the ear, psychophysical experiments were carried out with solutions of citric acid (sour), sodium chloride (salt) and sucrose (sweet), as well as with various other solutions The method of magnitude estimation was used and the results were plotted on log-log scales. In this way the power function exponents were obtained as the slopes of the observed straight lines. During the course of surgery performed on the middle ear, the investigators were able to measure the electrical responses in the exposed fibers of the chorda tympani to the application of these same solutions to the surface of the tongue. These data, also, were plotted on log-log scales. The power function exponents were found to be very similar to the corresponding exponents measured in the psychophysical experiments. Therefore, Borg *et al*. demonstrated, at least for the sense of taste, the legitimacy of using the same variable, F, with both the psychophysical and the neurophysiological interpretations.

We cannot, of course, generalize the above conclusions to include all other sensory modalities. For example, in the case of audition, the loudness of a tone cannot be mapped onto, or associated with the impulse rate in a single auditory neuron. We also see later that the time scale of neuronal events differs markedly from the time scale of psychophysical events.

We proceed, in the mathematical development that follows, as if each primary sensory afferent neuron functions independently and in parallel with all other primary sensory afferents, although we realize that this approximation cannot be taken too far. And we shall pretend, until Chapter 13, that subjective magnitudes always parallel the corresponding neural impulse rates as they do in the experiments of Borg *et al*. The mathematical work proceeds somewhat more fluently with these assumptions, but we understand that in the final analysis fuller recognition must be made of the distinction between psychophysics and neurophysiology. In the coming chapters, we usually treat F as a psychophysical variable because the experimental data available for testing the validity of our equations are much more numerous. I do, however, confess my uneasiness with the general process of assigning numbers, subjectively, to one's sensations. I use the results of these experiments involving subjective magnitudes because, at least at the beginning of our studies, they provide a convenient way of testing the theory. As the theory develops further, however, we shall work with experiments in which subjective magnitude plays a lesser role or no role at all.

Finally, let me stress that in the above analysis of the law of sensation we have ignored the variable, t, the time since onset of the stimulus. *Time* is a poor relative in papers describing experiments on subjective magnitudes; it is a variable that is always prominent in the conduct of the experiments, but is often not reported by the investigators. We treat t as a constant in these experiments. That is, we assume in all experiments, that the same period of time has elapsed between the onset of the stimulus and the measurement of F. If this were not the case, the effects of adaptation (see below) would wreak havoc. We also assume that new stimuli have been applied only after the sensory receptor has had time to recover completely from previous stimuli; that is, we presume that the sensory receptors are "unadapted." Please note, therefore, that the law of sensation is obtained from the three cardinal variables, F, I and t, by holding t constant and relating F to I.

We have also assumed tacitly, in the above discussion, that a unique algebraic form of the psychophysical law exists and governs many of the modalities of sensation. There are those who maintain that a unique psychophysical law is chimerical; that none exists. For example, Weiss (1981) argues, quite properly, that the algebraic form of any psychophysical law must depend on the manner in which the physical intensities, I, are measured. If we decided to measure the concentrations of odorants using the pC scale (the negative logarithm of the concentration of the odorant), then the psychophysical law, both the Fechner and the Stevens forms, would be patently wrong. Weiss is quite right. However, we shall show later on that there is a condition with which all measures of stimulus intensity must comply if they are to give rise to a universal psychophysical law: namely the logarithm of stimulus variance must be a linear function of the logarithm of stimulus magnitude: that is, $\sigma^2 \propto I^n$. This rule will be complied with (approximately) when concentrations of odorants or solutions are measured on linear scales, but not if they are measured on logarithmic or other scales. My reasons for affirming its existence will become clearer as we proceed. We shall return to Weiss' objection in Chapter 10.

ADAPTATION

The term *adaptation* seems to mean different things to different people. I use the term in two senses, the first of which is best introduced by example. Suppose you walk into a room dominated by the pungent odor of fresh paint, or perhaps into a kitchen enveloped by the heavy odor of cabbage cooking. In either case the odor is very prominent when you first enter the room, but weakens with time, and after a few moments may become virtually undetectable. We say that you have *adapted* to the olfactory stimulus. The phenomenon is often quite dramatic. When you first experienced the adaptation effect as a child did you not think that the stimulus itself had vanished? Certainly some adults I have spoken with still believe this to be true. “After a few minutes, the paint doesn’t smell any more,” a house-painter once told me. However, the stimulus is certainly still present; only our sensation of it has diminished. This type of adaptation might be called *psychophysical adaptation*.

It is important to observe that not all modalities of sensation seem to adapt, or if they do adapt, do so incompletely. The rate of adaptation may also vary considerably. Olfactory receptors, of course, do adapt and often adapt completely or *to extinction*. That is, odor simply disappears after a short period. Taste receptors adapt, but not necessarily to extinction; that is, the sweet taste from the sugar solution in your mouth may become less intense, but will not disappear completely. Temperature receptors behave in much the same fashion. Mechanoreceptors are classified by the speed with which they adapt, and their speed of adaptation specializes them as velocity receptors, vibration receptors, etc. (Schmidt 1978). Pain receptors do adapt to some extent. As you can imagine, a lot of work has been done on this subject. However, often pain persists and the sufferer requires chemical analgesia. Light receptors may not seem to adapt under normal circumstances. For example, the page you are reading is not fading. However, when you step from a dark room into bright sunlight, you may be temporarily blinded. After a few moments, the eye does light adapt and the world seems less bright (in a very literal sense!). Of course, we do not stay adapted forever; after the stimulus has been removed for a period of time, we “de-adapt” so that, for example, we may re-enjoy the scent of hydrogen sulphide gas.

The second meaning which I import to the term *adaptation* is *increase in threshold*. A *threshold* for sensation is the stimulus of least intensity which one can detect. Thresholds tend to increase as adaptation proceeds, and to decrease as de-adaptation occurs. For example, if you wish to detect visual stimuli of very low intensity, flashes of light consisting of only a few photons, a good strategy is to remain in a very dark room for about 30 minutes. The eye de-adapts (usually referred to as *dark adaptation*) so that the retinal photodetectors called *rods* can decrease their threshold of detection. The change in light intensity threshold (*luminance threshold*), ΔI, has been related to the *adapting luminance*, I, by Weber’s law (Dowling, 1987):

$$\Delta I \,/\, I = \text{constant.} \tag{3.1}$$

So the two meanings to be associated with adaptation are *psychophysical* and *threshold shift*. By and large, I shall be dealing with psychophysical adaptation. I shall regard adaptation as a phenomenon (again) involving two of the three cardinal variables. In the case of psychophysical adaptation, intensity, I, is held constant while the perceptual variable, F, changes with the time since stimulus onset, t. By and large, the psychophysical adaptation curve has the shape illustrated in Figure 3.2.

We recall that the law of sensation was regarded as the mathematical relationship between F and I with t held constant. Now, taking the effects of adaptation into account, we see that for many modalities, F will be be smaller when t is larger. Therefore, in a full logarithmic plot (power law of sensation) of F vs. I, the straight line obtained will shift downward on the graph when t is greater. For

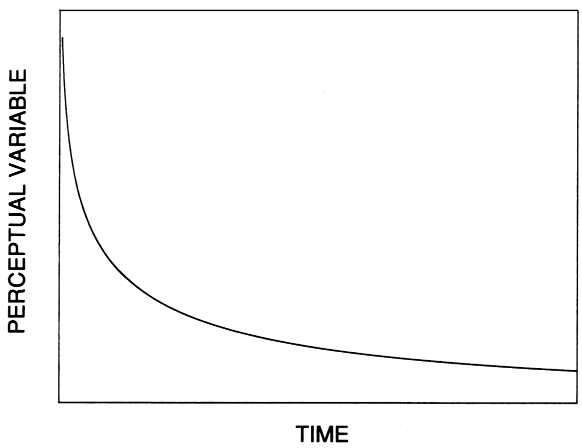

Figure 3.2 Psychophysical adaptation curve (schematic). A perceptual variable (e.g. magnitude estimate) declines monotonically with the time since stimulus onset. Stimulus intensity is held constant.

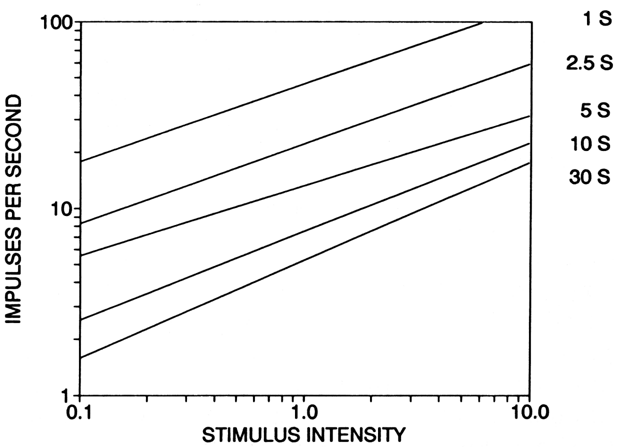

Figure 3.3 After Schmidt, 1978. Neural response of a pressure receptor on a log-log plot. Each straight line represents the "law of sensation" for a receptor at a specified time of adaptation (1 s, 2.5 s, ...). Clearly, the greater the adaptation time, the greater the downward shift of the straight line. We shall understand later why the lines are nearly parallel.

example, the neural response of a pressure receptor (impulses per second) to a constant-force stimulus (newtons) results in a series of nearly parallel straight lines (Schmidt, 1978, page 88). This effect is illustrated in Figure 3.3. The reason why the straight lines are parallel will emerge as we conduct our theoretical analysis.

The subject of auditory adaptation requires some further remarks. While there seems to be near-universal agreement that psychophysical adaptation does occur (for example, apparently pulsating tones are more easily detected than are steady tones), the extent of this adaptation is not completely clear (to me). However, there are two representations of auditory adaptation that are quite clear. The first is very rapid neurophysiological adaptation in the guinea pig auditory nerve reported by Yates *et al.* (1985) for the guinea pig auditory nerve. These investigators recorded the response of guinea pig ganglion cells to 100 ms tone bursts, in the form of peristimulus and poststimulus time histograms. The result was a rapid decline in the frequency of action potentials during the first 50 ms following the start of the tone stimulus. There is also a classical paper by Galambos and Davis (1943) purporting to show pronounced adaptation in single auditory nerve fibers of the cat. However, in a later note (1948), the authors queried their own work suggesting that the electrical effects measured may have issued from other neurons.

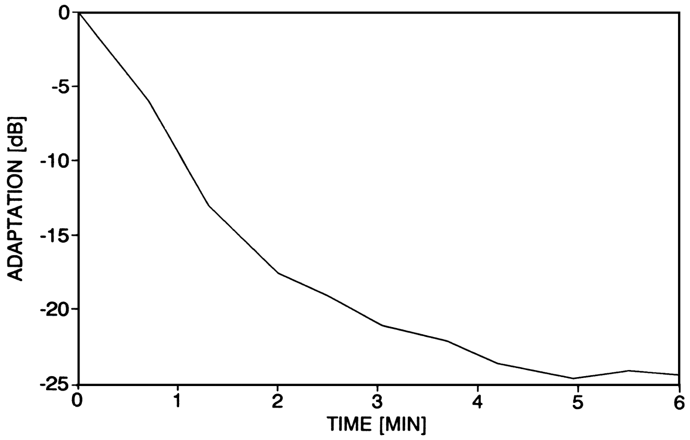

Figure 3.4 SDLB (simultaneous dichotic loudness balance) data of Small and Minifie (1961, Figure 3c, p. 1030). One ear, stimulated by means of a steady tone, adapts with respect to the other ear, which is stimulated only intermittently.

A second very clear manifestation of auditory adaptation, this time in the human being, was discovered by Hood† in 1950. Hood's method involves the comparing of loudness in one ear, which is being adapted to a tone, with the opposite ear, which is receiving very little sound. Specifically, this is how the effect is measured. A tone of constant intensity is presented to the *adapting ear*. It is this ear which is being tested for a decrease in loudness sensation. An intermittent tone is presented to the opposite or *test ear*. The subject must adjust the intensity of the tone in the test ear to balance the loudness between both ears. It is found that the adapting ear adapts with respect to the test ear. Note that in this clever measure of adaptation, the investigator does not record the subjective magnitude, or loudness of the tone, but rather records the objective or physical magnitude of the adaptation process. That is, he or she records the decrease in intensity of sound in decibels[2] required to produce a balance between ears. A graph of dB adaptation vs. time, using the data of Small and Minifie (1961) is given in Figure 3.4. It may be seen that about 30 dB of adaptation are recorded when one ear is tested with respect to the other. This process is called the technique of *simultaneous dichotic loudness balance*, or SDLB. Note that it, too, relates two of the three cardinal variables: I, the physical intensity of the tone (giving rise to dB of adaptation) and t, the time since onset of the tone. We shall study the adaptation process theoretically in Chapter 11.

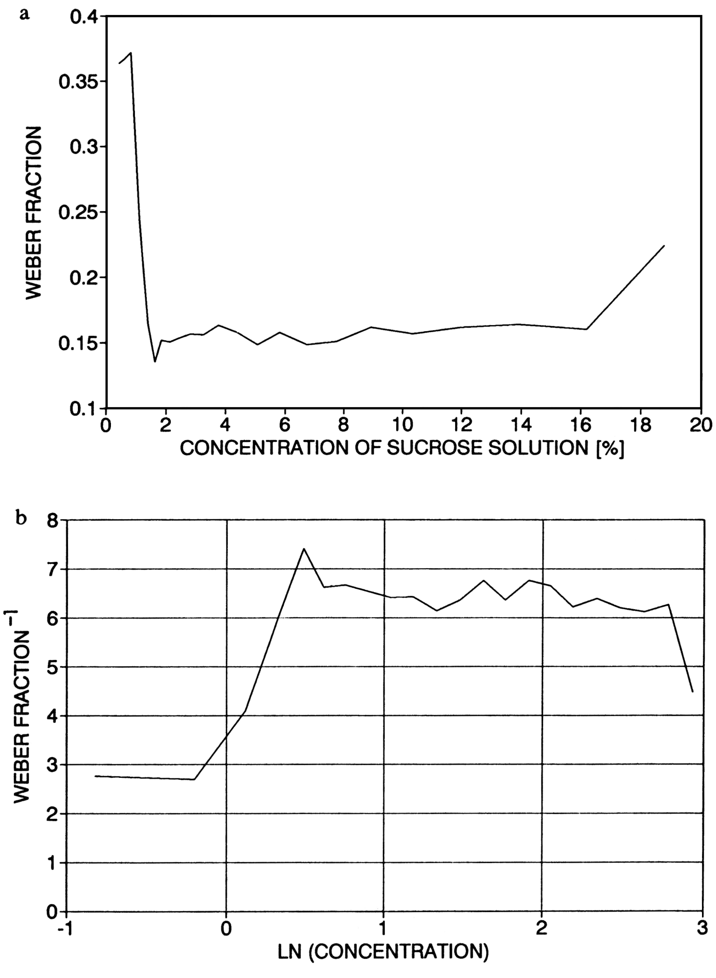

Figure 3.5 a&b (a) Data of Lemberger (1908) for differential threshold of taste of sucrose. Weber fraction plotted against concentration of tasted solution. Note the features of the curve: fall in Weber fraction for low intensities to a plateau region that extends from about 2% to 16% solution, followed by a terminal rise as the physiological maximum is approached. Lemberger actually provided three additional data points showing that for very high concentrations Weber fractions become very great (discrimination is poor as maximum concentration is approached).
(b) Same data as in (a). Inverse Weber fraction is plotted against ln (concentration). The number of rectangles beneath the curve between concentrations *a* and *b* is equal to the number of jnd's between *a* and *b*.

THE WEBER FRACTION

We have already encountered Weber's fraction, $\Delta I / I$, which is usually measured as the ratio of the smallest detectable difference between two stimuli, to the lower of the two stimuli. ΔI has also been called the *differential threshold* or *limen for intensity discrimination*. We saw in the formulation of Equation (3.1) that Weber and Fechner believed that $\Delta I / I$ was constant over the physiological range of perceptible I. However, later work by other investigators showed this not to be the case. For most, if not all, modalities of sensation the Weber fraction is maximum for the lowest intensities and falls progressively as I increases. For the middle range of intensities, $\Delta I / I$ is nearly constant, approximating Weber's law (3.1). For high values of I, approaching the maximum (non-painful) level of stimulation, $\Delta I / I$ again rises for many modalities. This terminal rise in the Weber fraction is certainly found for the sense of taste (Lemberger, 1908; see Figure 3.5a), and the sense of vision (König and Brodhun, as reported by Nutting, 1907, Table 1; and Hecht, 1934, Figure 3.6), and possibly also for the sensation of temperature (Pütter, 1922). High-intensity rise in the Weber fraction may also be a feature of auditory intensity discrimination (McConville *et al.* 1991), although this is far from certain.

Again, although most authors do not report the duration of stimuli used in measurements of intensity discrimination, it will be assumed that the duration is kept constant for all stimuli. Thus, the graph of $\Delta I / I$ vs. I is obtained by holding t, one of the three cardinal variables, constant, and plotting the relationship between the other two variables. The reader may object that only one of the remaining two variables is evident, namely I; the other variable, F, is not present. Actually, the variable, F, *is* present but is just not visible. Recall that ΔI refers to the change in intensity, I, that corresponds to one just noticeable difference in sensation. This jnd in sensation can be written in the form ΔF. Therefore, the expression for Weber fraction, if written out more fully would be

$$\frac{\Delta I}{\Delta F} / I ,$$

the change in stimulus intensity per jnd divided by the lower of the two intensities.

The experimental process whereby the Weber fraction is measured is of interest to us. There is not one single definitive value of ΔI for which a distinction between signal intensities can be made. Rather, ΔI must be inferred statistically from the experimental data. The experimental protocol might be as follows. The subject might be presented with two stimulus signals sequentially, and required to indicate which of the two was more intense. The difference between intensities can be designated δI. This procedure might be repeated many times with the same two stimuli, sometimes with the more-intense stimulus presented first and sometimes with the less-intense first. The proportion of correct discriminations, C, can then be computed. But C is a function of the difference in intensities, δI. That is, C is $C(\delta I)$. In order to define $C(\delta I)$ over a range of values of δI, the experiment can be repeated with

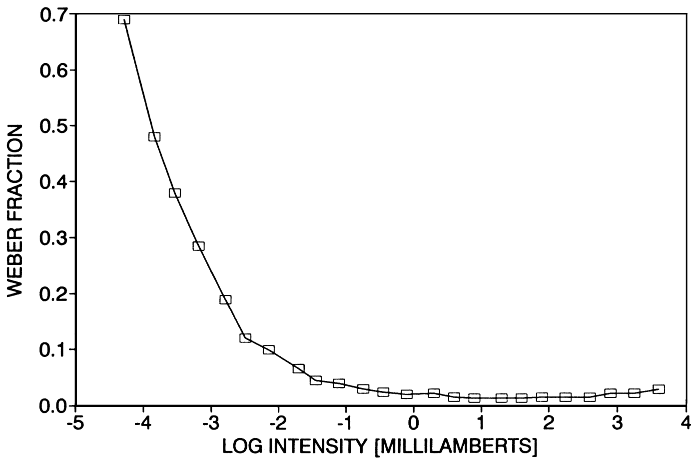

Figure3.6 Data of König for differential threshold of light intensities. Weber fraction is plotted against intensity of light. Manifests the same features as the corresponding curve for taste (Figure 3.5a).

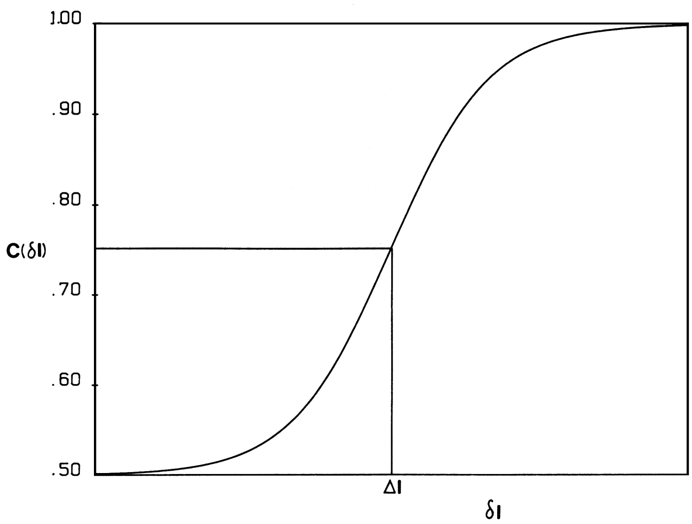

Figure 3.7 Statistical measurement of the differential threshold, ΔI Proportion of correct discriminations, $C(\delta I)$, plotted against intensity difference, δI. Define ΔI as the value of δI for which $C(\delta I)$ equals 0.75.

the lower of the two stimulus signals at the same value as before, but with δI changed. $C(\delta I)$ can again be computed. This process is repeated until the function $C(\delta I)$ has been defined for a range of values of δI. The graph of C vs. δI often obtained is sigmoid in shape, as shown in Figure 3.7.

The value of δI to be taken as ΔI, "the" limen for intensity discrimination, is largely arbitrary, but ΔI is commonly taken as the value of δI for which C is equal to 0.75; that is, ΔI is the stimulus increment that will permit a correct discrimination between intensities 75% of the time. ΔI is, of course, a function of I, so it must be determined at all requisite stimulus intensity levels. As you can see, there is a lot of experimental work involved in the determination of one Weber fraction curve with I ranging from threshold to maximum physiological intensity. A method for measuring the difference limen that is, perhaps, more classical is described by Coren and Ward (1989, p33 - 35).

The purpose of including the above experimental protocol in this book is primarily to demonstrate the *arbitrariness* of the function $\Delta I(I)$ (ΔI as a function of I). If the criterion for discrimination is changed from 0.75 to 0.5, for example, the function $\Delta I(I)$ will change accordingly. Therefore – and this is a feature we shall draw upon later – ΔI, the limen for intensity discrimination, is not a unique quantity. If ΔI is to appear as a variable in a theoretically derived equation, its lack of uniqueness must be compensated for. We shall have to deal with this problem when it arises.

Let us turn our attention now back to the graph of ΔI vs. I, where I extends over the full range of physiological values, from threshold to the verge of pain. The extent of the full physiological range will vary considerably depending on the modality of sensation. For example, for the sense of taste, $I_{max}/I_{min} \simeq 10^2$, while for the sense of hearing, $I_{max}/I_{min} \simeq 10^{11}$ (or greater). The value of $\Delta I/I$ may approach (or exceed?) unity for values of I close to threshold, and descend to values nominally in the range 0.1 — 0.5 in the middle range of I. If $\Delta I/I$ tends toward a plateau or constant value in this middle region, the constant will be referred to as *Weber's constant*, with reference to Equation (3.1).

It is rare, but always noteworthy, when people succeed in deriving a sensory law from another, more basic law, without the addition of major assumptions: the apparent complexity of the universe is demonstrably diminished. In this regard, it is worthwhile to see how Ekman (1959) was able to derive an expression for the Weber fraction from a variant of the power law of sensation. Ekman added one additional constant, a, to the power law, Equation (3.7), to obtain the equation

$$F = k(I + a)^n \ . \tag{3.9}$$

It transpires that the constant a must be greater than zero (see Equation (3.10) below), which makes the interpretation of Equation (3.9) somewhat difficult. If we differentiate F with respect to I, we find

$$\frac{dF}{dI} = \frac{nF}{I + a} \ .$$

Introducing an approximation using finite differences and solving for the Weber fraction,

$$\Delta I / I = \frac{a\Delta F / nF}{I} + \frac{\Delta F}{nF} .$$

At this point, Ekman found it necessary to introduce an additional equation,

$$\Delta F = cF ,$$

which we have seen before in the derivation of the power law of sensation. Combining the last two equations we obtain

$$\Delta I / I = \frac{ac/n}{I} + c/n , \tag{3.10}$$

or simply

$$\Delta I/I = A/I + B , \tag{3.10a}$$

where A and B are constants greater than zero. More of the history of this equation is given in Chapter 12.

Equation (3.10) does, indeed, describe the shape of the Weber fraction, showing that it has large values for small values of the intensity, I, and that it descends toward a constant plateau value of B for larger values of I. Apparently, Fechner himself proposed a modification of Equation (3.1) to Equation (3.10). Notice that Equation (3.10) does not allow a high-intensity rise in the Weber fraction.

From the graph of $\Delta I/I$ vs. I, can we calculate the total number of jnd's, N, which, when stacked one on the other, would extend from threshold to maximum physiological intensity? In principle, N can be measured directly (see Lemberger, 1908) by making N measurements; however, in practice, this is often impossible and one must calculate N from fewer than N measurements. N may be calculated from the equation,

$$N = \int_{I_{\text{thresh}}}^{I_{\text{max}}} \frac{dI}{\Delta I} . \tag{3.11}$$

The idea is that $dI/\Delta I$ is the number of jnd's that "fit into" a small intensity range, dI. If we then integrate from the value of intensity at threshold, I_{thresh}, to the maximum physiological value of I, I_{max}, we shall obtain N, the total number of jnd's.

Since ΔI has been measured for a number of intensities, I, we can calculate $1/\Delta I$ for these intensities. Equation (3.11) states that N is equal to the area of the curve formed by plotting $1/\Delta I$ against I for the full range of I. One can then plot the graph and find the area by numerical integration using, for example, the trapezoidal rule or Simpson's rule (see, for example, Press *et al.*, 1986). If I has a large range of values, such as in the senses of vision and hearing, Equation (3.11) may be difficult to employ, and I recommend a minor modification. Since

$$d(\ln I) = dI/I ,$$

$$dI / \Delta I = \frac{d(\ln I)}{\Delta I/I}$$

and Equation (3.11) may be written in the form

$$N = \int_{\ln I_{\text{thresh}}}^{\ln I_{\text{max}}} \frac{d(\ln I)}{\Delta I/I} . \tag{3.12}$$

That is, N is equal to the area of the curve obtained by plotting the reciprocal of the Weber fraction, $(\Delta I/I)^{-1}$, against the natural logarithm of I. Such a graph has been made from Lemberger's data in Figure 3.5b, and the reader can easily estimate the area by counting the number of large squares beneath the curve (about 21 jnd's).

If a plateau-region exists in the Weber fraction curve, say between the intensity levels I_{low} and I_{high}, then Equation (3.12) can be used to calculate, in a very simple way, the total number of jnd's, N_{plateau}, between these limits. Since $\Delta I \,/\, I =$ Weber's constant in this plateau-region, this quantity may be removed from under the integral sign:

$$N_{\text{plateau}} = \frac{1}{\text{Weber constant}} \int_{\ln I_{\text{low}}}^{\ln I_{\text{high}}} d(\ln I) \tag{3.13}$$

$$N_{\text{plateau}} = \frac{1}{\text{Weber constant}} \ln(I_{\text{high}} / I_{\text{low}}) \ ,$$

or

$$N_{\text{plateau}} = \frac{\ln\ 10}{\text{Weber constant}} \log_{10}(I_{\text{high}} / I_{\text{low}}) \ . \tag{3.14}$$

For example, referring to Figure 3.5b, if we approximate the plateau-region in Lemberger's data as extending between sucrose concentrations of 1 to 16, then, since the Weber constant $\simeq 0.14$,

$$N_{\text{plateau}} = \frac{\ln(16/1)}{0.14} = 19.8 \text{ jnd's.}$$

One final word on Weber fractions. When dealing with the sense of hearing, the variable, *I*, is sometimes used to represent mean sound pressure, *p*, and sometimes to represent mean sound intensity which varies as p^2. Therefore,

$$\Delta I / I = \Delta p^2 / p^2 \simeq 2p\, \Delta p / p^2 = 2 \Delta p / p \ . \tag{3.15}$$

Notice also the power law for sound intensity:

$$F = kI^n \rightarrow kp^{2n} \ , \tag{3.16}$$

so that the power function exponent for sound pressure is twice that for sound intensity.

THE ANALOGS

Let us digress briefly from our review of sensory experiment to consider again the ideal gas analog introduced in Chapter 1. Recall the three state variables, *P*, *V* and *T* and the three equations involving these variables derived from the ideal gas law, Equation (1.1):

$$P \propto T \qquad \textit{Charles' law}$$

$$P \propto 1/V \qquad \textit{Boyle's law} \tag{1.3}$$

$$\Delta T / T \propto 1/T \ . \tag{1.4}$$

To obtain Equation (1.2) we held *V* constant; to obtain Equation (1.3) we held *T* constant; and to obtain Equation (1.4) we again held *V* constant and considered the result when ΔP was also constant.

Compare the ideal gas equations with the psychophysical experiments that we have been discussing. Our three variables are now *t* (time since stimulus onset), *I* (intensity of stimulus) and *F* (perceptual variable). To obtain the law of sensation, we held *t* constant and obtained a graph where *F* increases monotonically with *I* (Figure 3.1; cf. Equation (1.2)). To study adaptation phenomena we held *I* constant and obtained a graph where *F* decreased monotonically with *t* (Figure 3.2; cf. Equation (1.3)). To study difference discrimination we again held *t* constant and found for constant ΔF that $\Delta I / I$ varied as a function of *I* (Figure 3.5; cf. Equation (1.4)).

$$\begin{array}{ccc} V & T & P \\ \Updownarrow & \Updownarrow & \Updownarrow \\ t & I & F \end{array}$$

The primary reason for introducing the *PVT* analogs is to aid (psychophysicists and biologists primarily) in regarding the three sensory experiments not as independent entities but rather as different experiments performed with the same three variables: I, t and F. Once this conceptual leap has been made, one has less difficulty understanding how a single equation, analogous to $PV = RT$, can serve to *unite* the three types of experiment. And unification is basically what this book is about.

Just as $PV = RT$ can be written as $P = P(T, V)$, the hypothetical unifying sensory equation can be written formally as

$$F = F(I, t)\ , \tag{3.17}$$

where the explicit form of the function, $F(I, t)$, has yet to be developed. When t is held constant (that is, $t = t' =$ constant), then

$$F = F(I, t') \tag{3.18}$$

will describe the law of sensation (presumably in both the full logarithmic and the semilogarithmic forms). When I is held constant (that is, $I = I' =$ constant), then

$$F = F(I', t) \tag{3.19}$$

will describe adaptation phenomena. When both t and ΔF are held constant (that is, $t = t'$ and $\Delta F = \Delta F'$), then

$$\Delta I / I = g(t', \Delta F'; I) \tag{3.20}$$

will describe the Weber fraction; g is some function yet to be defined. In the coming chapters we work toward the derivation of the critical function $F(I, t)$. We can, actually, be a little more explicit even at the present time. Since we have introduced the relationship

$$F = kH\ , \tag{2.6}$$

k constant, in Chapter 2, we know, therefore, that the critical function, F, can be expressed in the form

$$F(I, t) = k\, H(I, t)\ . \tag{3.21}$$

Our problem is, then, to derive the algebraic form of the functions $H(I, t)$.

THRESHOLD EFFECTS: THE LAWS OF BLONDEL AND REY, OF HUGHES, OF BLOCH AND CHARPENTIER

We move forward in time, now, from the mid-nineteenth century (Weber and Fechner) to the late nineteenth and early twentieth century. In 1885, Bloch and Charpentier stated their law governing the minimum quantity of light energy required for detection by an observer. In separate papers published in *Comptes Rendus de la Société de Biologie*, they argued that I_{thresh}, the minimum perceptible light intensity, is a function of the duration, t, of the light signal. In fact, for values of t less than about 0.1 second

$$I_{\text{thresh}} \cdot t = \text{constant}. \tag{3.22}$$

That is, the simple arithmetic product of I_{thresh} with t is constant. Since I_{thresh} can be measured in units of power (e.g. joules per second), the Bloch-Charpentier constant represents a minimum energy for signal detection. However, when t exceeds some upper bound, the law is violated. There is a minimum value for I_{thresh} below which no light stimulus is perceptible.[3] Let us call this value I_∞. Then

$$I_{\text{thresh}} \ge I_\infty\ . \tag{3.23}$$

The same law seems to hold for the sense of hearing, although I am not quite sure who first observed it. A graph published recently in the Handbook of Perception and Human Performance (Scharf and Buus, 1986) contains the collective data of Garner (1947), Feldtkeller and Oetinger (1956), and Zwislocki and Pirodda (1965). This graph shows that the threshold shift (in decibels)[2] of 1000 Hz tone bursts is a linearly decreasing function of the logarithm of time, for t less than 0.3 seconds. That is,

$$10\log_{10}(I_{\text{thresh}}/I_\infty) = -k\log_{10}t + \text{constant}, \qquad k > 0\,.$$

If one measures the value of k from the graph, it is found that k is very nearly equal to 10. That is, representing the constant on the right-hand side of the above equation by $10\log_{10}a$, we obtain to a good approximation

$$I_{\text{thresh}}/I_\infty = a/t$$

or

$$I_{\text{thresh}} \cdot t = aI_\infty\,, \tag{3.24}$$

Which is, again, the Bloch-Charpentier law. a can be estimated from the data to be about 0.16 second.

Moving forward a little in time to 1912, Blondel and Rey addressed the issue of the mathematical relationship between I_{thresh} and t for larger values of t; that is, for values of t greater than that for which the Bloch-Charpentier law was valid. The empirical relationship they discovered, which has been confirmed by many other studies, is the following:

$$\frac{I_{\text{thresh}}}{I_\infty} = 1 + \frac{a}{t}\,. \tag{3.25}$$

The constant a, usually now known as the Blondel-Rey constant, has a value of about 0.21 seconds. As Blondel and Rey pointed out, when t, the duration of the stimulus, is small, the second term on the right-hand side of Equation (3.25) becomes much greater than one, and consequently

$$\frac{I_{\text{thresh}}}{I_\infty} \simeq \frac{\alpha}{t}\,,$$

which retrieves the Bloch-Charpentier law. Equation (3.25) is, therefore, a more general empirical equation embracing the earlier law.

I have found the proceedings of a symposium on flashing lights chaired by J. G. Holmes (1971) to be a valuable source of information on this subject.

Not to be outdone by the vision researchers, J. W. Hughes (1946) published his research on the auditory threshold of brief tones, and showed that the Blondel-Rey equation was valid also for tones of various frequencies between 250 and 4000 Hz. Hughes drew the readers' attention to the similarity between Equation (3.25) and the equation giving the threshold electrical current which passes through a nerve cell membrane, as a function of the time needed to achieve threshold or firing of the neuron (the "chronaxie equation of Lapicque"). Hughes does not seem to give sufficient data in his paper for the evaluation of the constant, a, for audition. However, since again the equation goes over into the Bloch-Charpentier law for brief t, one might estimate a to have the value of about 0.16 second (see Equation 3.24).

I am not aware of Hughes' work having been replicated, but Plomp and Bouman (1959) have extended Hughes' studies.

SIMPLE REACTION TIME

The subject's finger is poised above a button that will register the exact time it is pressed. On her ears are headphones. The instant she hears a tone through the headphones she will press the button. The time between the beginning of the tone and the pressing of the button is called the simple reaction time. In general (Coren and Ward, 1989), the reaction time is the time between the onset of a stimulus (auditory, visual, gustatory, ...) and the subject's overt response.

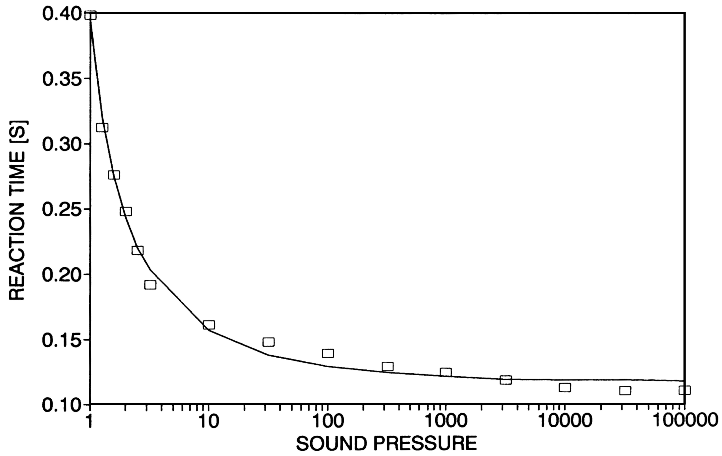

Figure 3.8 Data of Chocholle (1940). Reaction time plotted against sound pressure. The smooth curve is discussed later in the text.

A feature that makes the simple reaction time particularly interesting is its peculiar relationship to the intensity of the stimulus: The more intense the stimulus, the shorter the reaction time. This relationship between reaction time, t_r, and stimulus intensity is shown in Figure 3.8 (Chocholle, 1940). There is, of course, a threshold intensity below which the stimulus cannot be detected.

There are various physiological events that transpire during the reaction time. A neuronal signal must travel from the sensory receptor(s) to the brain passing one or more synapses, a motor signal must proceed down to muscle, and muscle must contract to actuate the finger. The study of these components dates back at least to the time of Helmholtz. We shall return later to muse, briefly, over this sequence of events.

We see, though, that on the whole, the relation between reaction time and stimulus intensity is, again, a relationship between the two variables, I, and t. However, $t = t_r$ is not necessarily the duration of the stimulus (the sense in which t has been used before), but rather is the time taken by the subject to react to the stimulus. This will lead us later into somewhat darker waters.

Simple reaction time, too, has its associated empirical equations, the best-known of which are probably those of Piéron. Although Piéron formulated several empirical relations between t_r and I, the one with which we shall be most concerned is the following (Piéron, 1914, 1952):

$$t_r = t_{r\,\min} + CI^{-n} \tag{3.26}$$

where C and n are constants that are greater than zero, and $t_{r\,\min}$ is the smallest possible value of t_r, obtained for the maximum physiological value of I. It can be seen that Equation (3.26) describes the type of curve depicted in Figure 3.8. Moreover, an extraordinary observation has been made, particularly for simple reaction times to auditory and visual stimuli: the value of the exponent, n, in Equation (3.26) is close to, but usually less than, value of n found in the power law of sensation,

$$F = kI^n\ . \tag{3.7}$$

Why in the world should this be so? Is it pure coincidence?

We demonstrate later, using the unifying equation (3.21) in its explicit form, that both Equation (3.7) and Equation (3.26) can be derived from the same "parent" equation, and that the exponent, n, can be expected to be similar in magnitude in both equations.

However, despite all that we shall attempt to do, "simple" reaction time will retain some secrets that we are not able to fathom.

Just a general remark here before proceeding. It should be recognized that all of the preceding empirical equations represent means or averages taken on many, many trials involving many individual subjects. There are large inter-subject differences and no attempt has been made, in these pages, to tabulate them. Neither shall we be able, in the theoretical exposition that follows, to make allowance for these differences. Rather, we are content just to be able to derive the equations for the means.

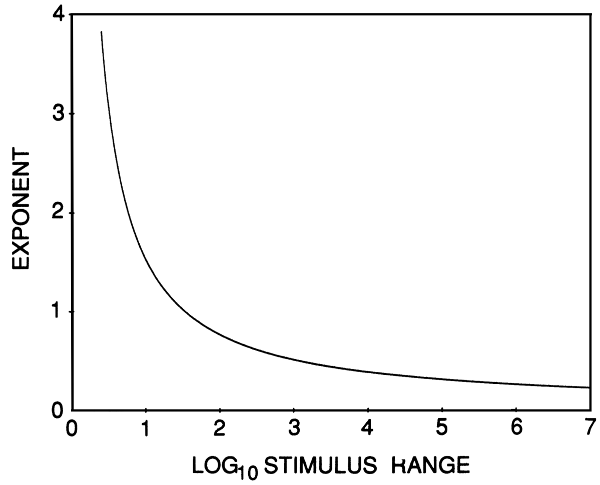

Figure 3.9 Schematic of graph by Teghtsoonian using data assembled by Poulton. Power function exponent plotted against $\log_{10}$ stimulus range = 1.53. The data array themselves along a rectangular hyperbola.

THE POULTON-TEGHTSOONIAN LAW

In a well-known paper published in 1971, R. Teghtsoonian, working with data assembled by E. C. Poulton (1967), made an extraordinary observation. He observed a relationship between the power function exponents, n (Equation 3.7), for different sensory modalities and the logarithm of the physiological range spanned by these modalities. For example, audition has the exponent value of about 0.3 (sound intensity at 1000 Hz.), and auditory intensity spans a range of about 10^9, so that $\log_{10}(\text{range}) \simeq 9$. The sense of taste gives rise to an exponent much closer to 1.0, while it spans only about 2 decades of concentrations (intensities), so that $\log_{10}(\text{range}) \simeq 2$. Higher exponents are associated with a smaller range and vice versa.[4] In fact, when Teghtsoonian plotted a graph of n vs. $\log(\mathit{range})$ (shown schematically in Figure 3.9), he found that the data lay on a rectangular hyperbola whose equation was

$$(n)(\log_{10}\text{range}) = 1.53\ . \tag{3.27}$$

We shall see, in our explorations, that we can derive Equation (3.27) in the course of our theoretical studies of the Weber fraction.

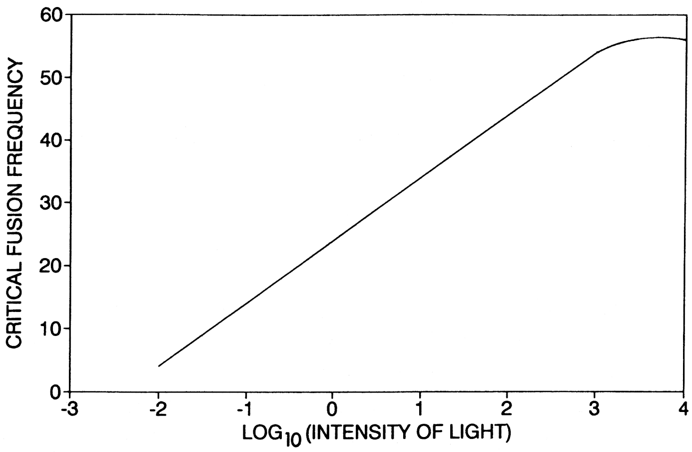

Figure 3.10 Schematic demonstration of the Ferry-Porter law. Critical fusion frequency for a flickering light source is related linearly to the logarithm of the intensity of the light, to a saturable limit. The curve drawn is characteristic of the type obtained when the observed light falls on the fovea.

A VERY APPROXIMATE LAW OF OLFACTORY THRESHOLDS

The final empirical equation we shall discuss here was discovered by Laffort *et al*. (1974), and elaborated by Wright (1982). It is a law that holds only very approximately, but may be worth mentioning here anyway.

Let I_∞ be the lowest detectable concentration of an odorant. Now define

$$p_{\mathrm{ol}} = -\log_{10} I_\infty \,. \tag{3.28}$$

Laffort *et al*. discovered a hyperbolic relationship between n and p_{ol} quite similar to Equation (3.27):

$$(n)(p_{\mathrm{ol}}) = \text{constant}. \tag{3.29}$$

This relationship has not always been confirmed by other investigators.

THE FERRY-PORTER LAW AND TALBOT'S LAW

The final empirical law we shall discuss here was formulated by Ferry (1882) and Porter (1902) and deals with flashing lights. It refers to an experiment in which a subject is observing a flashing light, or a rotating disk with black and white sectors. The frequency of the flashing light is held constant. Let us suppose that the on-time of the light is equal to the off-time. The frequency of flashing is slowly increased. At a certain frequency the subject reports that the light no longer appears intermittent but rather appears to be steady. The frequency of flashing may then be slowly decreased until the light again appears intermittent. In this way, a *critical fusion frequency* or *critical flicker frequency* (*CFF*) for the light of a given wavelength for a given intensity is established. The experiment may then be repeated for a number of different intensities of light, keeping the wavelength constant.

It is found that the critical fusion frequency increases with increasing intensity of the light, up to a certain maximum intensity. The shape of the graph of *CFF* vs. I is influenced by the regions of the retina on which the image of the light falls, and also shows the effects of the two types of light receptors, rods and cones, that are found in the human retina. However, *CFF*, over a wide range of intensities, is found to increase linearly with the logarithm of the intensity (Figure 3.10). That is,

$$CFF = a \log I + b, \qquad a, b \text{ constant}. \tag{3.30}$$

This semilogarithmic law is called the *Ferry-Porter law*.

One should notice here again that, as with so many of the preceding empirical laws of sensation, we deal with a relationship between the intensity of the stimulus, and the duration of time over which the stimulus is applied (the on-time of the light in each cycle).

At frequencies that exceed the critical fusion frequency, the effective luminance (intensity) of a flashing light is independent of frequency and is equal to the average over time of the real luminance. This phenomenon is called *Talbot's law*.

With this law we conclude our tour of the empirical laws of sensation. We have not, by any means, exhausted the stock of such laws; many, many more of them exist. However, all eight laws are dealt with in a theoretical sense in the course of this book.

NOTES

1. A word to the wise... What Weber actually wrote in Latin is "in observando discrimine rerum inter se comparatarum, non differentiam rerum, sed rationem differentiae ad magnitudinem rerum inter se comparatarum, percipimus" (cited by Drever, 1952).

2. Decibels (dB):

Let x and x_0 be any two numbers. We can use x_0 as a reference with respect to which x is reported. For example, we could report all values of x as multiples of x_0:

$$q_1 = x_1 / x_0, \qquad q_2 = x_2 / x_0 \ldots$$

The decibel system reports x as a logarithm of the q's. For example,

$$\mathrm{dB}_1 = 10\log_{10}q_1 = 10\log_{10}x_1/x_0\ ;$$
$$\mathrm{dB}_2 = 10\log_{10}q_2 = 10\log_{10}x_2/x_0\ ;\ \ldots$$

In general

$$\mathrm{dB} = 10\log_{10}x/x_0\ .$$

Why do we use the dB-system? Why not just report x as x? Sometimes the values of x we are interested in become very large or very small; for example $x_1 = 10^{-7}$, or $x_2 = 10^{8}$. It is then convenient to choose a handy value of x_0 (say $x_0 = 1$) and use the dB-system. Then

$$x_1 = 10\log_{10}10^{-7} = -70\ \mathrm{dB}$$
$$x_2 = 10\log_{10}10^{8} = 80\ \mathrm{dB}.$$

Notice that you can always use the dB-value to solve backwards to obtain the original x-value. For example, what x-value corresponds to y dB?

$$y = 10\log_{10}x$$
$$\log_{10}x = y/10$$
$$x = 10^{y/10}\ .$$

3. Modern signal detection theory addresses the problem of threshold detection probabilistically, but we shall not introduce SDT in this book.

4. The values given for "range" are low. There is an arbitrary element involved in specifying the upper limit of the range.

† (2003 ed. note) I had not realized, at the time of writing, that von Bekesy had suggested essentially the same test many years before Hood. The reader is referred to *Experiments in Hearing* by Georg von Bekesy, published by the Acoustical Society of America, through the American Institute of Physics, by arrangement with McGraw-Hill Book Company, which published the book originally in 1960. Bekesy's experiment appears on page 357 of the book. Moreover, Bekesy was prescient in his use of the log-scale for time, which is suggested now by the entropy theory (Equation (11.45)).

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